Ministry of Education and Science of the Russian Federation

National Research University – Novosibirsk State University (NRU NSU)

Department of Mechanics and Mathematics

MASTER EDUCATIONAL PROGRAMME
FOR TEACHING FOREIGN STUDENTS IN ENGLISH

Probability and Statistics

Qualification (degree) of the graduate
Master of Mathematics

Full-time tuition

Supervisors: **LOTOV Vladimir I.**, Professor of the Chair of Probability and Statistics NRU NSU, Chair head
CHERNOVA Natalia I., Associated Professor of the Chair of Probability and Statistics NRU NSU

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Introduction

National Research University – Novosibirsk State University. Department of Mechanics and Mathematics

Novosibirsk State University (NSU; http://www.nsu.ru/exp/index.jz?lang=en) was established in 1958. NSU was growing up together with world known Novosibirsk Scientific Center (Akademgorodok) focusing on training highly qualified specialists for science and education.

The Department of Mechanics and Mathematics (DMM; http://www.nsu.ru/exp/en/education/mechanics_and_mathematics) is one of the leading divisions of NSU. It was founded in 1961. The DMM has three levels of education: Bachelor degree, Master degree and postgraduate study. Students get trained in the following fields: Mathematics, Mathematics and Computer Science, Applied Mathematics and Information Science, Mechanics and Mathematical Modeling.

The main courses on DMM are taught by outstanding scientists from different research institutes of Siberian Branch (SB) of Russian Academy of Sciences (RAS). Besides attending the courses, students have an opportunity to join leading scientific schools in these institutes. Both professors and students take an active part in cooperative projects along with researchers from the leading universities and scientific organizations of Russia and the whole world. Senior students who work in RAS research institutes carry out research in modern fields of mathematics and its applications (including various fundamental and modern issues of mathematical model construction and computer science). Such specialization diversity allows students to prepare for carrying out research in scientific institutes as well as for practical work in many areas applying the latest information technologies.

Every year the International Student Scientific Conference "Students and the Progress in Science and Technologies" takes place in NSU (http://issc.nsu.ru/index.php?lang=1). Here students and young scientists give talks on their researches and get acquainted with the results of their colleagues.

At 2009, after the competition held among the leading Russian universities, the Novosibirsk State University won the category National Research University (NRU). This category was awarded by the Government of the Russian Federation on the basis of special Development Program of NSU for the period 2009-2018 (http://www.nsu.ru/exp/en/university/national_research_university). The strategy goal of the Program is (quote): *"to form a research-educational system of national and global importance…which will be able to provide an advised specialist training based on science, education and business integration"*.

At 2013 NRU NSU became one of fifteen Russian universities that had won the right to get a subsidy favoring the university advancement in the world ratings.

Chair of Probability and Statistics DMM NRU NSU

The Chair of Probability and Statistics was founded in 1965. The Chair was headed by Prof. A.A. Borovkov from 1965 to 1973 and from 1979 to 2012, and by Prof. B.A. Rogozin from 1973 to 1979. Since 2012 the Chair Head is Prof. V.I. Lotov. The work of the Chair is closely connected with the activity of the Laboratory of Probability and Mathematical Statistics of the Sobolev Institute of Mathematics, the Siberian Branch of the Russian Academy of Sciences. Almost all Chair members are also on the laboratory staff. So one may speak of a united team of researchers and Chair members representing a scientific school that has won worldwide recognition. This school was established by Prof. A.A. Borovkov, the member of the Russian Academy of Sciences, who has been providing leadership since the very foundation. Professor Borovkov is an outstanding scientist who has a worldwide reputation for his fundamental contributions to various areas of Probability and Statistics such as Limit Theorems for Stochastic Processes, Boundary Problems, Asymptotically Optimal

Procedures in Mathematical Statistics and Queueing Theory. He is also the author of several monographs and textbooks on Probability Theory and Mathematical Statistics.

The following well-known specialists in Probability and Statistics were the chair members: professors S.V. Nagaev, B.A. Rogozin, S.A. Utev, V.V. Yurinskii, L.Ya. Savel'ev. Pupils of this team now work in many other scientific centers in Russia and abroad. Currently, the members of the Chair are well-known mathematicians: Professors I.S. Borisov, S.G. Foss, D.A. Korshunov, V.I. Lotov, A.A. Mogul'skii, A.I. Sakhanenko, Associated Professors A.A. Bystrov, E.A. Baklanov, N.I. Chernova, A.P. Kovalevskii, Yu.Yu. Linke, P.S. Ruzankin. All of them are the Siberian School graduates.

The principal fields of activity of the Chair are the following: reading core and elective courses on the Probability Theory and Mathematical Statistics, training of highly qualified specialists in these areas, and research on a wide range of problems in Probability and Statistics.

The teaching process is closely related to research activity. The Chair is involved in a number of international and Russian scientific projects, and all the researchers and post-graduate students take part in this work. For more than 40 years, two scientific seminars are functioning: the research seminar and the reviewing seminar. Their participants are graduate students as well as scientists from Sobolev Institute of Mathematics and their colleagues from other universities and scientific centers. The Chair members participate in many international scientific meetings and are invited as the lecturers at foreign universities. Every year the Chair organizes the section “Probability and Statistics” at the International Student Scientific Conference “Students and the Progress in Science and Technologies”. Chair staff serve as members of the editorial boards of Russian and foreign mathematical journals such as “Theory of Probability and Its Applications”, “Siberian Mathematical Journal”, “Queueing Systems”, “Siberian Advances in Mathematics”, “Siberian Journal of Industrial Mathematics”, and others.

The main directions of scientific activity of the Chair staff are related to the following fundamental areas of Probability Theory and Mathematical Statistics:

– Limit Theorems and their refinements, including convergence in infinite dimensional spaces;
– Analysis of Large Deviation Probabilities;
– Boundary Problems for Random Walks and their applications;
– Asymptotic Methods in Mathematical Statistics;
– Ergodicity and Stability of Stochastic Processes;
– Mathematical Models of Queueing and Communication Networks.

Development of the Master Educational Programme (MEP) “Probability and Statistics”

The Development Program of NRU NSU for the period 2009-2018 implies introduction of individual educational pathways on the basis of the University Master Training Center and organization of English-speaking training groups. A certain contribution to development of such Center is the elaboration of the Master Educational Programme (MEP) “Probability and Statistics” for teaching foreign students in English. This MEP allows foreign student to get Russian state diploma at the end of education because it complies the State Master Standard (SMS) for training direction 010100 – “Mathematics” (approved by the Order № 40 of Ministry of Education and Science of the Russian Federation on January, 14; 2010, see http://fgosvpo.ru/uploadfiles/fgos/30/20110325143133.pdf [Russian]).

Probability is a core mathematical discipline, alongside geometry, algebra, and analysis. In recent years, the evident power and utility of probabilistic reasoning as a distinctive method of scientific inquiry has led to an explosive growth in the importance of probability theory in scientific research. Central to statistics and commonplace in physics, genetics, and information theory for many decades, the probabilistic approach to science has more recently become indispensable in many other disciplines, including finance, geosciences, medical sciences, artificial intelligence and communication networks. Probability and statistics are used to model uncertainty from a variety of sources, such as incomplete or simplified models. Statistical tools are at work in almost every area of life, including agriculture, business, engineering, medicine, law, regulation, and social policy, as well as in the physical, biological, and social sciences and even in parts of the academic humanities. Last decades the scope of probability theory and statistics increased with the emergence of new sub-fields such as queueing theory and renewal theory, nonparametric statistics, time series analysis, sequential analysis.

In most of these areas deep results have been obtained by the leading researchers of Siberian probability school. The lecture courses of the MEP "Probability and Statistics" will include the latest advances in all these areas.

The MEP also includes required courses of the SMS 010100 – "Mathematics" and some disciplines from the international master programmes "Numerical Statistical Modelling and Simulation. Monte Carlo Methods" and "Modern Trends in Discrete Mathematics and Combinatorial Optimization" of Department of Mechanics and Mathematics NRU NSU.

The MEP "Probability and Statistics" is the essentially updated and revised version of the master programme of the same title in Russian, which was elaborated in 2010 (authors – Professor V.I.Lotov, Assoc. Prof. N.I.Chernova; scientific supervisor – Academician A.A.Borovkov).

Information about the authors of the Programme

Personal Data

Lotov Vladimir Ivanovich (http://www.math.nsc.ru/LBRT/v1/lotov/lotov.html) was born in 1949. He graduated from the Chair of Probability and Statistics of NSU in 1971 with Master Thesis "Limit Properties of the Concentration Functions", scientific supervisor — prof. B.A.Rogozin. In 1977 V.I.Lotov received Ph. D. degree at Steklov Mathematical Institute, Moscow, Russia for the work "Asymptotic Expansions in Two-sided Boundary Crossing Problems for Random Walks" supervised by prof. B.A.Rogozin. And in 1989 V.I.Lotov achieved D.D. (Doctor of Science, or Research Doctorate degree) at Steklov Mathematical Institute for the Thesis "Limit Theorems in Two-sided Boundary Crossing Problems for Random Walks". Now V.I.Lotov is chief scientist at the laboratory of the Probability Theory and Mathematical Statistics of the Sobolev Institute of Mathematics and Chair Head at NSU.

His main research interests include Boundary Crossing Problems for Random Walks, Factorization Methods, Limit Theorems and Asymptotic Expansions for Distributions of Boundary Functionals, Sequential Analysis. V.I.Lotov has about 95 scientific papers published in Russian and international journals and several tutorials for NSU students. For more than 35 years he was a lecturer and conducted classes at undergraduate level in Probability and Statistics for students in Mathematics, Physics, Economics, Biology and Information Technologies, taught classes at graduate level in Boundary Crossing Problems for Random Walks, Sequential Analysis at Dept. of Probability and Mathematical Statistics (for more than 35 years). He has supervised of more than 40 undergraduate and 4 graduate students.

Chernova Natalia Isaakovna (http://www.nsu.ru/mmf/tvims/chernova/index.html) was born in 1966. She graduated from the Chair of Probability and Statistics of NSU in 1989, and received Ph.D.

degree in 1996 with Thesis “Ergodic Theorems for Complex Stochastic Queueing Networks” supervised by Prof. S.G.Foss. From 1989 to the present she is working at the Chair of Probability and Statistics of NSU. Main research interests of N.I.Chernova are in the area of Queueing Theory: Ergodic Properties, Monotonicity and Stability of Queueing Systems and Networks. N.I.Chernova has about 10 papers in Russian and international scientific journals and several tutorials for students.

Lists of recent publications of V.I.Lotov and of N.I.Chernova

1. V.I.Lotov. On the limit behavior of the distribution of the crossing number of a strip by sample paths of a random walk. Siberian Math. J., 2013,V. 54, Iss.2, pp. 265-270.

2. K.V. Lotov, G. Z. Lotova, V. I. Lotov, A. Upadhyay, T. Tuckmantel, A. Pukhov, and A. Caldwell. Natural noise and external wakefield seeding in a proton-driven plasma accelerator. Physical Review Special Topics - Accelerators and Beams, 2013, V. 16, Iss. 041301, 6 p. http://prst-ab.aps.org/abstract/PRSTAB/v16/i4/e041301

3. V. I. Lotov, A. S. Tarasenko. On the distribution of the first exit time and overshoot in a two-sided boundary crossing problem. Siberian Advances in Mathematics, 2013, V. 23, Iss. 2, pp. 91-98.

4. V.I.Lotov. On the asymptotics of weighted renewal function. J. of Mathematical Sciences, 2013, V. 188, Iss. 4, pp. 435-440.

5. Irle A., Lotov V. On the properties of a multiple sequential test. Metrika, 2010, V. 72, Iss. 2, pp.189-198.

6. V. Lotov. On the sojourn time of a random walk in a strip. Siberian Math. J., 2010, V. 51, Iss. 4, pp. 621-638.

7. V.I. Lotov. Factorization identities for the sojourn time of a random walk in a strip. Siberian Math. J., 2010, V. 51, Iss. 1, pp. 119-127.

8. V.I.Lotov, N.G.Orlova. Asymptotic expansions for the distribution of the crossing number of a strip by a Markov-modulated random walk. Siberian Math. J., 2006, V. 47, Iss. 6, pp. 1066-1083.

9. Lotov V.I. On the mean value of the ladder epoch for random walks with a small drift. Izvestiya: Mathematics, 2006, V. 70, Iss. 6, pp. 1225-1232.

10. Irle A., Lotov V.I. A nonsymmetric test. Metrika, 2004, v. 59, No. 2, 137-146.

11. Natalia Chernova, Sergey Foss, Bara Kim. A polling system whose stability region depends on a whole distribution of service times. Oper. Res. Letters, 2013, V. 41, Iss. 1, pp.188-190.

12. Natalia Chernova, Sergey Foss, Bara Kim. On the Stability of a Polling System with an Adaptive Service Mechanism, Annals of Oper. Res., 2012, V. 198, p. 125-144.

13. S. G. Foss, N. I. Chernova. On optimality of the FCFS discipline in multi-server queueing systems and networks. Siberian Math. J., 2001, V. 42, Iss. 2, pp. 372–385.

14. S. Foss, N. Chernova. On Stability of a Partially Accessible Multi-Station Queue with State-Dependent Routing. QUESTA, 1998, V. 29, Iss. 1, pp. 55–73.

Brief description of the MEP “Probability and Statistics”

Definition of the Master Educational Programme (MEP)

The ***Master Educational Programme (MEP) “Probability and Statistics”*** is a system of educational documents based the State Master Standard (SMS) for training direction 010100 – “Mathematics”, specialization "Theory of Probability and Mathematical Statistics" (approved by the Order № 40 of Ministry of Education and Science of the Russian Federation on January 14, 2010) and the requirements for graduates of magistracy of Department of Mechanics and Mathematics (DMM) of National Research University – Novosibirsk State University (NRU NSU). The MEP includes curriculum, programs of courses, conditions for admission to the MEP and acquired knowledge and skills of graduates.

Aims and objectives of the MEP

The purpose of development of Master Educational Programme is methodical support of the above mentioned SMS 010100 for the specialization in Probability Theory and Mathematical Statistics and of the requirements of NRU NSU for the training of highly qualified specialists in this field.

The main goal of the MEP is to train highly qualified specialists who are able to solve new problems and to produce new knowledge in various fields of Probability Theory and Statistics, to apply probabilistic and statistical results in models arising in manufacturing and finance, in computer networks and quality control, in risk assessment and reliability.

The key objectives of education are

- to acquaint with methodology of Probability Theory and Mathematical Statistics as a method of studying natural phenomena
- to give theoretical knowledge of probabilistic and statistical models and tools which are widely used in physics, economical and natural sciences
- to develop the skills of scientific inquiry
- to develop the urge to obtain and to produce new knowledge
- to train the ability to communicate the results of scientific research to the audience
- to develop the ability to accept criticism and to analyze the results of their own work.

MEP gives knowledge in stochastic processes, random walks, risk theory, financial mathematics, martingales, regression analysis, applied statistical methods, Markov chains, multivariate limit theorems, asymptotic properties of functionals of order statistics, as well as in fundamentals of modern numerical methods theory and of modelling and simulation.

Student graduated in MEP can be employed in science, education, industry or continue their education in graduate school.

Training period of the MEP

The training period of the MEP “Probability and Statistics” in full-time education is 2 years. One academic year includes two semesters (“fall” and “spring” semesters), two test and examination sessions (“winter” and “summer” sessions), see below the section “Sample curriculum of the MEP”. Two winter holidays and one summer holidays are also provided.

An academic year begins at September, 1. The fall semesters (the first and the third semesters of the MEP) proceed 18 weeks: from September, 1 till the end of December. The winter test sessions (after the first and the third semesters of the MEP) occur at the end of December. The winter examination sessions proceed in January.

The winter holidays (after the first and the third semesters of the MEP) proceed one week at the beginning of February.

The second spring semester proceeds 16 weeks: from February till the end of May. The second summer test and examination sessions proceed in June. The summer holidays (after the second semester of the MEP) proceeds two months: from July, 1 till August, 31.

The fourth “spring” semester proceeds 14 weeks: from February till May. The fourth summer test and examination sessions proceed at the end of May. Then (in June) the procedures of final state certification (including the defending the master thesis) are carried out.

Training capacity of the MEP

Training capacity for training within the MEP “Probability and Statistics”, including all kinds of classroom and independent work, research student’s work and the time taken for quality control, is 120 credits (units of study); one credit (unit) is equal to 36 academic hours – see below the section “Sample curriculum of the MEP”.

Conditions for admission to the MEP "Probability and Statistics"

Educational level of the entrants

The MEP "Probability and Statistics" is designed to teach both foreign students and graduates of the Russian higher education institutions (including bachelor graduates of NRU NSU).

The entrant to the MEP should have an educational diploma (certificate or some analogous document) on programme of level of a bachelor degree in the field of mathematics. This programme must include an advanced mathematical course (courses) with basic elements of mathematical and functional analysis, linear algebra, mathematical physics, probability theory and mathematical statistics, numerical mathematics and programming. Desirable knowledge of English language: TOEFL Internet based equivalent score 50-70 (intermediate or upper intermediate). Some knowledge of Russian language (for scientific and household communication) is also useful (but not necessary).

For passing of the programme the Russian entrants can be accepted as well (including graduates of NRU NSU). For Russian entrants the submission of the bachelor's degree diploma is obligatory.

Selection procedure to the MEP (the major dates)

Till ***May, 15*** the applicant sends the following copies of documents by electronic mail to the supervisor of the program Prof. Lotov Vladimir Ivanovich (e-mail lotov@math.nsc.ru) or to Dr. Natalia Chernova (e-mail: cher@nsu.ru):

1. Diploma (or some analogous document) on programme of level of a bachelor degree or certificate (ordering) about passing such programme at the moment.
2. TOEFL certificate (score 50-70: intermediate or upper intermediate level).
3. Recommendation of the professor of mathematics or applied mathematics.
4. Biography of the applicant.
5. Motivational letter (1-2 pages) on entrance to the MEP.

Till ***June, 1*** the procedure of selection of applicants for testing passing is carried out.

Till ***June, 10*** the interview with committee of leading professors of the Chair of Probability and Statistics NRU NSU is carried out (in internal form or by Skype). Features of this interview are reflected in the following section of the programme.

Till ***June, 25*** the Russian entrants (and wishing foreign applicants; except bachelor graduates of DMM NRU NSU having the special admission to the magistracy – see the site http://mmf.nsu.ru/applicants/magistracy [in Russian]) additionally pass the admission test. The examples of admission test are shown in the Appendix 2 of this programme; see also the site http://mmf.nsu.ru/applicants/master-entexams [in Russian]**).**

Till ***July, 1*** the admittance to the MEP occurs.

Interview on the Chair of Probability and Statistics NRU NSU

The interview with committee of leading professors of the Chair of Probability and Statistics NRU NSU includes control questions and simple tests on basics of mathematical and functional analysis, linear algebra, mathematical physics, probability theory and mathematical statistics. Examples of interview control questions are presented in the Appendix 2.

In case the committee makes the decision on acceptance of the applicant, the entrant's scientific supervisor (one of the professors of the Chair of Probability and Statistics NRU NSU) is nominated. This person defines the choice of training courses, the direction of scientific research for preparation of term and dissertations works during training time of the MEP. The committee can also determine an individual additional leveling educational list of basic courses (in the case when the applicant shows insufficiently profound knowledge during the interview).

Formal reception to the MEP. Training payment

In case of successful passing of interview, the testing committee of the Chair of Probability and Statistics NRU NSU formulates the written recommendation for the entrant to the MEP (this document must be signed by the Head and Secretary of the Chair and also by the nominated scientific supervisor of the entrant) and sends it to dean's office of Department of Mechanics and Mathematics (DMM) NRU NSU.

The training for master on DMM NRU NSU is paid (in 2014-2015 the corresponding price is equal to 5200 dollars for one year). For the students having a high rating, possibility of receiving a state or NSU grant is provided (www.russia.edu.ru, http://www.nsu.ru/exp/university/oms/magistratura and http://www.nsu.ru/exp/magistratura/pravila_priema [in Russian]).

Learning outcomes of the MEP "Probability and Statistics"

General comments

MEP "Probability and Statistics" aimed to produce a qualified specialists in the field of Theoretical Probability and Mathematical Statistics. The Program graduates become able to deal with various stochastic models, analyze and solve actual applied problems using probabilistic approaches, to understand existing and to prove new results in modern areas of Probability and Statistics and to create corresponding scientific reviews, reports and papers.

The graduates of the MEP can choose the following kinds of activity:

– *postgraduate study of NRU NSU, scientific institutions of Siberian Branch of RAS (in particular, in IM SB RAS) and other universities around the world;*
– *research work in laboratories of NRU NSU and in scientific institutions of SB RAS;*
– *teaching in higher education institutions:*
– *work in research divisions of factories and firms;*
– *business and advisory activity.*

Acquired knowledge and skills (according to the MEP curriculum)

In accordance to the SMS for training direction 010100 – "Mathematics" the MEP "Probability and Statistics" includes the following cycles and parts (see below the section "Sample curriculum of the MEP"):

– *M.1-B General scientific cycle, the base part;*
– *M.1-V General scientific cycle, the variable part;*
– *M.2 Professional cycle, the main (variable) part;*
– *M.3 Scientific practice and research work;*
– *M.4 Final state certification.*

In ***the section M.1 "General scientific cycle",*** the competences ***GCC-1-6, GCC-9-10*** and ***PC-1-2, PC-6-8, PC-10-15*** are formed. Resulting on this part, ***the MEP graduate student***
should be able to work in an interdisciplinary team (GCC-1);
should be able to communicate with experts from other fields (GCC-2);
should be ready to active social mobility, to work in an international environment (GCC-3);
should have depth knowledge of the legal and ethical standards in the assessment of the consequences of his professional activity in the development and implementation of social projects (GCC-4);
should be able to generate new ideas (GCC-5);
should be able to work independently, to care about the quality, to strive for success (GCC-6);
should have the ability to organize and plan (GCC-9);
should have the ability to find, to analyze and to process information, including an information related to new areas of knowledge that are not directly related to the field of professional activity (GCC-10).
should know methods of mathematical modeling in the analysis of global issues on the basis of in-depth knowledge of fundamental mathematical disciplines and computer science (PC-1);
should know methods of mathematical and algorithmic modeling in the analysis of problems of natural science (PC-2);
should be able to build an adequate and complete image of Probability Theory and of Mathematical Statistics, and of other probabilistic disciplines (PC-6);
should be able to explore the modern computer algorithms, to improve, enhance and develop the mathematical theory behind their basis (PC-7);

should have its own view on the possible applied aspects in the rigorous mathematical formulations (PC-8);
should be able to identify the common shapes, patterns, tools for groups of disciplines (PC-10);
should be able to possess of the methods of mathematical and algorithmic modeling in the analysis of economic and social processes and of tasks arising in business, in financial and actuarial mathematics (PC-11);
should be able to choose the different ways to represent and to adapt the mathematical knowledge based on the level of audience (PC-12);
should be able to guide and to lead the scientific work groups (PC-13);
should be able to formulate the non-mathematical types of knowledge (including human) in the task form (PC-14);
should be able to teach physics, mathematics and computer science in secondary schools and universities (PC-15);

In ***the section M.2 – "Professional cycle, the main (variable) part",*** the competences ***GCC-5-6*** and ***PC-4, 7, 9, 10, 13*** are formed. Resulting on this part of study, ***the MEP graduate student***
should be able to generate new ideas (GCC-5);
should be able to work independently, to care about the quality, to strive for success (GCC-6);
should be able to independently recognize and to analyze the physical aspects of a classical forms of mathematical problems (PC-4);
should be able to explore the modern computer algorithms, to improve, to enhance and to develop the mathematical theory behind their basis (PC-7);
should be able to apply creatively, to develop and to implement complex mathematical algorithms in modern software systems (PC-9);
should be able to identify the common shapes, patterns, tools for groups of disciplines (PC-10);
should be able to guide and lead the scientific work groups (PC-13).

In ***the section M.3 "Scientific practice and research work"*** the competencies ***GCC-1-10*** and ***PC-1-3*** are formed. In accordance to the SMS for training direction 010100 – "Mathematics", ***the MEP graduate student***
should be able to work in an interdisciplinary team (GCC-1);
should be able to communicate with experts from other fields (GCC-2);
should be ready to active social mobility, to work in an international environment (GCC-3);
should have depth knowledge of the legal and ethical standards in the assessment of the consequences of his professional activity in the development and implementation of social projects (GCC-4);
should be able to generate new ideas (GCC-5);
should be able to work independently, to care about the quality, to strive for success (GCC-6);
should have skills and abilities in the organization of scientific-research and scientific-production work in the management of research teams (GCC-7);
should possess initiative and a desire to lead (GCC-8);
should have the ability to organize and plan (GCC-9);
should have the ability to find, to analyze and to process information, including an information related to new areas of knowledge that are not directly related to the field of professional activity (GCC-10).
should know methods of mathematical modeling in the analysis of global issues on the basis of in-depth knowledge of fundamental mathematical disciplines and computer science (PC-1);
should know methods of mathematical and algorithmic modeling in the analysis of problems of natural science (PC-2);
should be able to intensive research and scientific and exploration activities (PC-3).

As a result of ***the section M.4 "Final state certification", the MEP graduate student***
should be able to publicly present their own research results (PK-5);
should be able to build an adequate and complete image of Probability Theory and of Mathematical Statistics, and of other probabilistic disciplines (PC-6);
should be able to extract relevant scientific and technical information from electronic libraries, reference journals (PC-16).

Professional characteristics of the MEP graduates

The area of professional activity of graduates according to the MEP "Probability and Statistics" meets SMS for training direction 010100 – "Mathematics" and includes research activities in the fields of science which use mathematical methods and computer technology; the solution of various problems using mathematical modeling, facilities and software; development of effective methods for solving problems of natural science, engineering, economics and management; information support of scientific research, engineering design, operational and management activities; teaching cycle of mathematical sciences (including computer science).

The objects of professional activity of graduates of the MEP "Probability and Statistics" are the concepts, hypotheses, theorems, methods and mathematical models that make up the content of fundamental and applied mathematics, mechanics, and other natural sciences.

Professional activities of graduates of the MEP "Probability and Statistics" conform to SMS for training direction 010100 – "Mathematics" and the requirements for the graduates graduate of Mechanics and Mathematics Department of NIU NSU. Master of direction "Probability and Statistics" is ready to deal with the following types of professional activity: research and exploration; production and processing; organization and management; teaching.

The tasks of professional activity of graduates of the MEP "Probability and Statistics" meet the requirements of SMS for training direction 010100 – "Mathematics". For instance, a graduate should be able to address the following professional tasks in accordance with the directions of professional activities:

1) Research and exploration activities:
- an application of mathematical and algorithmic modeling in the study of real-world objects and processes in order to find effective solutions to the general scientific, organizational and practical problems;
- analysis and synthesis of the results of scientific research in the field of mathematics with the use of modern science and technology, advanced domestic and foreign experience;
- preparation and holding of seminars, conferences and symposia;
- preparation and editing of scientific publications;
- the use of basic mathematical tools and mathematical methods in research;

2) Production and technological activities:
- the use of basic mathematical knowledge and creative skills to adapt quickly to new challenges arising in the process of computing and mathematical methods, to increasing of the complexity of the algorithms and mathematical models, to the needs in rapid decision-making in new situations;
- the use of modern computer technologies and software for statistical data analysis, and for analysis of network models and queueing systems, or other probabilistic models and algorithms;
- accumulation, analysis and systematization of the information required with the use of modern

methods of automated data collection and processing;
- methodological development of regulatory documents and participation in determining the strategy of development of corporate network;

3) Organizational and administrative activity:
- the application of science to predict the performance, quantitative and qualitative assessment of the consequences of decisions;

4) Teaching activity:
- lectures, seminars and other forms of educational process in the fields of probability theory, mathematical statistics, stochastic processes and queueing theory in higher education, the teaching of other physical of mathematical sciences and computer science in secondary schools and universities.

Structure and features of the MEP “Probability and Statistics”

Sample curriculum of the MEP

Index	Discipline title	Credits	In total hours	Class-room work	Indepen-dent work	1 sem.	2 sem.	3 sem.	4 sem.	Sertifica-tion form
M.1	**General scientific cycle**	**26**	**936**	**13**	**13**	**8**	**8**	**10**	**0**	
M.1-B	**Basic part (core courses)**	**20**	**720**	**9**	**9**	**6**	**6**	**8**		
M.1-B-1	Philosophy	4	144	2	2		2	2		Test
M.1-B-2	History of Mathematics	2	72	1	1			2		Test
M.1-B-3	Stochastic Processes	6	216	3	3	4	2			Exam
M.1-B-4	English academic writing	4	144	2	2	2	2			Test
M.1-B-5	Operations research (**)	4	144	2	2			4		Ex/test
M.1-V	**Variable part** (elective courses)	**6**	**216**	**3**	**3**	**2**	**2**	**2**		
M.1-V-1	Monte Carlo methods (basic course) (*)	2	72	1	1	2				Ex/coll.
M.1-V-2	Foreign (Russian) Language	4	144	2	2	2	2			Test
M.1-V-3	Number Theory	2	72	1	1			2		Exam
M.1-V-4	History of Numerical Statistical Modelling and Simulation (*)	2	72	1	1			2		Test
M.1-V-5	Inverse problems (*)	2	72	2	2		2			Exam
M.1-V-6	Combinatorial Optimization (**)	2	72	1	1			2		Test
M.1-V-7	Stochastic models of meteorological processes (*)	2	72	1	1		2			Exam
M.1-V-8	Modern problems of numerical mathematics (*)	2	72	1	1		2			Exam
M.2	**Professional cycle, the main (variable) part**	**30**	**1080**	**15**	**15**	**8**	**8**	**6**	**8**	
M.2-C	**Core courses**	**12**	**432**	**6**	**6**	**4**	**4**	**2**	**2**	
M.2-C-1	Advanced Probability	6	216	3	3	4	2			Exam
M.2-C-2	Martingale Theory	4	144	2	2			2	2	Exam
M.2-C-3	Random Walks	2	72	1	1		2			Exam
M.2-E	**Elective courses**	**18**	**648**	**9**	**9**	**4**	**4**	**4**	**6**	
M.2-E-1	Applied statistics	2	72	1	1	2				Test
M.2-E-2	Insurance Theory	2	72	1	1	2				Exam
M.2-E-3	Applied regression analysis	2	72	1	1			2		Test
M.2-E-4	Financial mathematics	2	72	1	1		2			Exam
M.2-E-5	Markov Chains	2	72	1	1		2			Exam
M.2-E-6	Theory of V-Statistics	4	144	2	2			2	2	Exam
M.2-E-7	Statistics of Stochastic Processes	2	72	1	1				2	Exam
M.2-E-8	Limit Theorems for Sums of Multivariate Random Variables	2	72	1	1				2	Exam
M.2-E-9	Asymptotic Analysis of Functionals of Order Statistics	2	72	1	1			2		Exam
M.2-E-10	Regression Analysis	2	72	1	1		2			Exam
M.2-E-11	Boundary Crossing Problems for Random Walks	2	72	1	1	2				Exam

Index	Discipline title	Credits	In total hours	Class-room work	Indepen-dent work	1 sem.	2 sem.	3 sem.	4 sem.	Sertifica-tion form
M.2-E-12	Queueing Theory	2	72	1	1	2				Exam
M.2-E-13	Advanced Stochastic Simulation Methods in Applied Mathematics and Physics (*)	4	144	2	2			2	2	Exam
M.2-E-14	Random Process Simulation and Continuous Stochastic Models (*)	2	72	1	1			2		Exam
M.3	**Scientific practice and research work**	**52**	**1872**	**12**	**40**	**14**	**14**	**14**	**10**	
M.3-1	Scientific seminar “Probability Theory and Mathematical Statistics”	8	288	4	4	2	2	2	2	Test
M.3-2	Reviewing seminar “Probability Theory and Mathematical Statistics”	8	288	4	4	2	2	2	2	Test
M.3-3	Preparation and oral defense of the term paper	8	288		8	4	4			Test
M.3-4	Research work in a laboratory of Probability and Mathematical Statistics of Sobolev Institute of Mathematics	24	864	4	20	6	4	10	4	Test
M.3-5	Reports at scientific conferences: International student scientific conference “Students and the progress in science and technologies” or others	4	144		4		2		2	
M.4	**Final state certification**	**12**	**432**	**2**	**10**				**12**	
M.4-1	Passing the State Exam	6	216	2	4				6	Grade
M.4-2	Preparation and defense of the Master Thesis	6	216		6				6	Grade
	TOTAL:	**120**	**4320**	**41**	**79**	**30**	**30**	**30**	**30**	

(*) – course of the MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”
(**) – course of the MEP “Modern Trends in Discrete Mathematics and Combinatorial Optimization”
Remarks.
1) This curriculum was prepared in accordance with SMS for training direction 010100 – “Mathematics”.
2) Term papers, the current control and interim evaluation (tests and exams) are regarded as a kind of academic work on the subject and are executed within the course hours.
3) In accordance with standard regulations of the university the types of training activities include: lectures, consultations, seminars, workshops, labs, tests, colloquiums, independent work, research work, practice, course work.

Semester programs of the MEP

THE FIRST (FALL) SEMESTER

Core courses (10 credits)

M.1-B-3 Stochastic Processes – the first part; 4 credits
M.1-B-4 English academic writing – the first part; 2 credits
M.2-C-1 Advanced Probability – the first part; 4 credits

Elective courses (min 6 credits)

M.1-V-1 Monte Carlo methods (basic course); 2 credits
M.1-V-2 Foreign (Russian) Language – the first part; 2 credits
M.2-E-1 Applied Statistics; 2 credits
M.2-E-2 Insurance Theory; 2 credits
M.2-E-11 Boundary Crossing Problems for Random Walks; 2 credits
M.2-E-12 Queueing Theory; 2 credits

Research work (14 credits)

M.3-1 Scientific seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-2 Reviewing seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-3 Preparation and oral defense of the term paper; 4 credits
M.3-4 Research work in the laboratory of Probability and Mathematical Statistics of Sobolev Institute of Mathematics; 6 credits.

TOTAL MINIMUM – 30 CREDITS

THE SECOND (SPRING) SEMESTER

Core courses (10 credits)

M.1-B-1 Philosophy – the first part; 2 credits
M.1-B-3 Stochastic Processes – the second part; 2 credits
M.1-B-4 English academic writing – the second part; 2 credits
M.2-C-1 Advanced Probability – the second part; 2 credits
M.2-C-3 Random Walks; 2 credits

Elective courses (min 6 credits)

M.1-V-2 Foreign (Russian) Language – the second part; 2 credits
M.1-V-5 Inverse Problems 2 credits
M.1-V-7 Stochastic models of meteorological processes; 2 credits
M.1-V-8 Modern problems of numerical mathematics; 2 credits
M.2-E-4 Financial Mathematics; 2 credits
M.2-E-5 Markov Chains; 2 credits
M.2-E-10 Regression Analysis; 2 credits

Research work (14 credits)

M.3-1 Scientific seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-2 Reviewing seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-3 Preparation and oral defense of the term paper; 4 credits
M.3-4 Research work in the laboratory of Probability and Mathematical Statistics of Sobolev Institute of Mathematics; 4 credits.
M.3-5 Reports at scientific conferences; 2 credits

TOTAL MINIMUM – 30 CREDITS

THE THIRD (FALL) SEMESTER

Core courses (10 credits)

M.1-B-1 Philosophy – the second part; 2 credits
M.1-B-2 History of Mathematics; 2 credits
M.1-B-5 Operations Research; *4* credits
M.2-C-2 Martingale Theory – the first part; 2 credits

Elective courses (min 6 credits)

M.1-V-3 Number Theory; 2 credits
M.1-V-2 History of Numerical Statistical Modelling and Simulation; 2 credits
M.1-V-6 Combinatorial Optimization; 2 credits
M.2-E-3 Applied Regression Analysis; 2 credits
M.2-E-6 Theory of V-Statistics – the first part; 2 credits
M.2-E-9 Asymptotic Analysis of Functionals of Order Statistics; 2 credits
M.2-E-13 Advanced Stochastic Simulation Methods in Applied Mathematics and Physics – the first part; 2 credits
M.2-E-14 Random Process Simulation and Continuous Stochastic Models; 2 credits

Research work (14 credits)

M.3-1 Scientific seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-2 Reviewing seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-4 Research work in the laboratory of Probability and Mathematical Statistics of Sobolev Institute of Mathematics; 10 credits.

TOTAL MINIMUM – 30 CREDITS (UNITS)

THE FOURTH (SPRING) SEMESTER

Core courses (2 credits)

M.2-C-2 Martingale Theory – the second part; 2 credits

Elective courses (min 6 credits)

M.2-E-6 Theory of V-Statistics – the second part; 2 credits
M.2-E-7 Statistics of Stochastic Processes; 2 credits
M.2-E-8 Limit Theorems for Sums of Multivariate Random Variables; 2 credits
M.2-E-13 Advanced Stochastic Simulation Methods in Applied Mathematics and Physics – the second part; 2 credits

Research work (22 credits)

M.3-1 Scientific seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-2 Reviewing seminar "Probability Theory and Mathematical Statistics"; 2 credits
M.3-4 Research work in the laboratory of Probability and Mathematical Statistics of Sobolev Institute of Mathematics; 4 credits
M.3-5 Reports at scientific conferences; 2 credits
M.4-1 Passing the State Exam; 6 credits
M.4-2 Preparation and defense of the Master Thesis; 6 credits

TOTAL MINIMUM – 30 CREDITS (UNITS)

Contents of courses and disciplines of the MEP "Probability and Statistics"

M.1 GENERAL SCIENTIFIC CYCLE
M.1-B BASIC PART

Title of the discipline:

M.1-B-1 Philosophy

Discipline description

The Philosophy represents one of the most time-honored disciplines of learning within the university setting. Addressing fundamental questions about truth, the nature of existence, human knowing, justice, ethical deliberation, and the experience of beauty, the study of philosophy cultivates within students an ability to read carefully and to think critically. Importantly, these questions concern not only human experience, but other academic disciplines as well. The course is devoted mainly to the Philosophy of Science and, in particular, of Mathematics. The Philosophy of Mathematics is the branch of Philosophy that studies the philosophical assumptions, foundations, and implications of Mathematics. The aim of the Philosophy of Mathematics is to provide the methodology of Mathematics and to understand the place of this science in people's lives.

Title of the discipline:

M.1-B-2 History of Mathematics

Discipline description

The course "History of Mathematics" is organized as a seminar by supervising of one of leading researchers of NRU NSU or IM SB RAS. All the participants prepare thematic reports on the development of mathematics and of specific areas of science. The course describe the main directions in development of mathematics and the history of its famous problems. The particular attention will be paid to mathematics and mathematicians of the 20th century. A special section will be devoted to the history of the Siberian Branch of the Russian Academy of Sciences and of Novosibirsk State University.

The most detailed is supposed to tell about the history of the development of different areas in Analysis and in Probability Theory. Supposed to talk about the problems related to the teaching of Probability Theory and of Mathematical Statistics. A special section will be devoted to the well-known problem and paradoxes.

Title of the course:

M.1-B-3 Stochastic Processes

Information about the author:

Ruzankin Pavel Sergeevich

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- Senior Researcher of the Laboratory of Probability Theory and Mathematical Statistics at the Sobolev Institute of Mathematics of Siberian Branch of RAS.
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Course description

Original lecture course Stochastic Processes was developed for teaching foreign students studying Probability and Statistics in English. This is a compulsory course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics” , “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability Theory and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - “Mathematics”. As a prerequisite, a modest understanding of Probability theory at an undergraduate level is assumed.

The main part of the course is concerned with the theory and applications of stochastic processes. The widely-spread classes and representatives of stochastic processes are discussed. Certain limit theorems are given. Some attention is given to basic classes of queueing systems.

The aim of the course “Stochastic Processes” is to acquaint students with actual theoretical aspects of stochastic processes theory, to develop students' skills in the use of probabilistic methods for solving mathematical problems as a part of research on the theory of stochastic processes, and in other areas of mathematics. To achieve this goal the following course objectives are formulated: to introduce students to the basic concepts and methods of the theory of stochastic processes, to give an idea of the current state and development of the area, to form students' skills in conceptual apparatus of the theory of stochastic processes.

Learning outcomes of the course

As the result of study of the course “Stochastic Processes” the student

should understand the role and place of this discipline among the other mathematical subjects;

should know the definitions, basic properties and examples related to the Stochastic Processes and their practical value;

should be able to apply their knowledge to solve mathematical problems.

Course content

1. Definition of a stochastic process. Distribution of a process. Sample probability space.
2. Markov chains with discrete time. Classification of states. Ergodicity of Markov chains.
3. Continuous-time Markov chains. Ergodicity. Kolmogorov’s differential equations. Generating matrix. Death and birth processes.
4. Processes with independent increments.

5. Poisson processes. Properties of trajectories.
6. Kolmogorov's theorem on existence of a continuous modification of a process.
7. Wiener processes. Continuity of paths. Non-differentiability of paths. The law of iterated logarithm.
8. Convergence of a sequence of Bernoulli processes to the corresponding Poisson process.
9. The functional central limit theorem (the Donsker-Prokhorov invariance principle).
10. Multi-dimensional Gaussian distributions, Gaussian processes.
11. Poisson point processes (spatial Poisson processes). Modelling of such processes.
12. Processes in L_2-space of random variables. Continuity and differentiability of these processes.
13. The Riemann integral for L_2-processes.
14. Elementary stochastic orthogonal measure. Stochastic integral of non-random function.
15. Ito's integral.
16. Simple stochastic differential equations.
17. M/M/1 queueing system. Convergence to the stationary distribution.
18. M/G/1 queueing system. The stationary distribution of the embedded Markov chain. The waiting times. The busy period.
19. G/M/1 queueing system. The stationary distribution of the embedded Markov chain. The waiting times.
20. G/G/1 queueing system. The asymptotic waiting time. The embedded random walk.
21. Closed migration processes. The equilibrium distribution.
22. Open migration processes. The equilibrium distribution.

Method of assessment

The program of the course provides for the following types of discipline assessment: two control tasks and the colloquium, the interim test control and oral exam.

Literature

1. Borovkov, A. A.: Probability Theory. Springer, 2013
2. Rozanov, Yu. A.: Itroduction to the theory of random processes. Nauka, 1982.
3. Korshunov, D. A., Voss, S.G., Eisymont, I. M.: Problems and exercises in probability theory. Lan, 2004.
4. Grimmett, G., Stirzaker, D.: Probability and Random Processes. Oxford University Press, 2001.

Title of the discipline:

M.1-B-4 English academic writing

Discipline description

Good mathematical writing in English is a skill which must be practiced and developed for optimal performance. The purpose of this discipline is to provide assistance for young mathematicians writing their first paper. The aim is not only to aid in the development of a well written paper, but also to help students begin to think about mathematical writing in English.

Training on this discipline will be organized in the form of special seminars under the chairmanship of leading researchers of Sobolev Institute of Mathematics SB RAS. Participation of members of editorial boards of "Theory of Probability and Its Applications", "Siberian Mathematical

Journal", "Siberian Advances in Mathematics", "Siberian Journal of Industrial Mathematics", "Queueing Systems" and others is supposed.

Learning outcomes of the discipline

As the result of discipline students gain skills for analyzing and writing mathematical texts in English.

Method of knowledge assessment

Term paper, research paper or a report at the student conference with the research results are considered as the final test of the course.

Basic literature

Kutateladze S.S. Russian → English in writing. Novosibirsk: Institute of Mathematics SB RAS, 2000 [in Russian]

Title of the course:

M.1-B-5 Operations research

(course of the MEP "Modern Trends in Discrete Mathematics and Combinatorial Optimization")

Information about the authors

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Course description

This course is designed to introduce students to basic models, concepts, and methods of Operations Research (OR). The course starts with brief introduction to computational complexity and description of some basic OR models. In the following discussion, some common methods to deal with optimization problems are observed (such as dynamic programming, enumeration methods) and several important graph optimization problems are considered (network flow problems, matching and assignment problems). Some "non-optimization" models (project planning and analysis, game-theoretic models) are considered as well. The course concludes with brief observation of some approximation and heuristics techniques. "Operations Research" course provides basic knowledge for more advanced courses of the Master's Program.

Learning outcomes of the course

At the end of the study the students *should know*:

1. the place of Operations Research among other branches of discrete and continuous mathematics

dealing with decision management and optimization;
2. a wide range of Operations Research models and problem formulations applicable in different fields of human activity;
3. algorithmic and computational aspects of OR models;
4. common methods and approaches to solve OR optimization and non-optimization problems.

At the end of the study the students *should be able to*:
- use the terminology common to the field of discrete mathematics, operations research and decision making;
- give full and correct proofs of mathematical statements;
- accurately translate real-world problems into adequate OR models;
- perform the complexity analysis of a problem, prove its NP-completeness/hardness;
- use different techniques to find exact and approximate solutions of optimization problems;
- investigate structural properties of optimization problems and devise new algorithms to solve them;
- analyse performance aspects of an algorithm (such as its time and space complexity, approximability, etc.), identify performance bottlenecks and give performance improvement recommendations.

At the end of the study the students *should possess*:
1. skills in formalizing and modeling a variety of decision-making processes;
2. skills in investigating discrete and continuous optimization problems;
3. skills in algorithm development and analysis.

Course Content

The lecture part of the course includes 17 lectures (34 hours overall) divided into 9 sections and 17 practical classes (34 hours overall) to students practice in applying algorithms and method described in the lectures. The detailed description is given below. Some particular themes can be replaced or modified by including some new results.

Lecture Course (34 hours)

Section 1. Introduction (2 hours)

1.1. Mathematical Models in OR.

Section 2. Computational (time) Complexity (4 hours)

2.1. Introduction to Computational Complexity Theory.
2.2. Using NP-completeness to analyze problems.

Section 3. Dynamic Programming (4 hours)

3.1. Bellman's Optimality Principle. Dynamic Programming.
3.2. Knapsack Problem. The Nearest Neighbor Problem.

Section 4. Project Planning (4 hours)

4.1. Project Network models and their characteristics.
4.2. Computation of the project's parameters.

Section 5. Implicit Enumeration (4 hours)

5.1. Branch and Bound method.
5.2. Balas additive algorithm.

Section 6. Matchings and Assignments (4 hours)

6.1. Matching and Vertex Cover. Maximum Matching in a bipartite graph.
6.2. Assignment Problem.

Section 7. Basics of Game Theory (4 hours)

7.1. Matrix Game. Pure Strategy and Nash Equilibrium.

7.2. Mixed Strategies.

Section 8. Network Flow Problems (4 hours)

8.1. Maximum Flow Problem.

8.2. Minimum Cost Flow Problem..

Section 9. Approximation Algorithms (4 hours)

9.1. Approximation Algorithms and their characteristics.

9.2. Heuristics and Metaheuristics.

Practical Course (34 hours)

1. Mathematical modeling in OR.
2. Computational complexity. Algorithm performance analysis. Polynomial reducibility and NP-completeness.
3. Proving NP-completeness and inapproximability results.
4. Dynamic Programming. The Shortest Path Problem. The Nearest Neighbor Problem.
5. Distribution of Effort Problem. Boolean Knapsack Problem (BKP). Inverse BKP.
6. Project Planning. Project network models. Network simplification.
7. The project's critical parameters. Bellman-Ford algorithm.
8. Branch and Bound for the Traveling Salesman Problem.
9. Balas additive algorithm for the Integer Linear Programming.
10. Maximum Matching Problem. The labeling algorithm.
11. The Assignment Problem. The Hungarian algorithm.
12. Matrix Games: the basic concepts. Pure-strategy Nash Equilibrium
13. Mixed Extension and Mixed-strategy Equilibrium. Methods of solving games.
14. Network Flows. Max-Flow Problem. Ford-Fulkerson algorithm.
15. Min-Cost Flow Problem. Klein's and Busacker-Gowen algorithms.
16. Approximation algorithms. Performance analysis.
17. Approximation schemes.

Method of assessment

In the course, the following types of formative assessment are used: questioning and homework (weekly), and computational assignment (semester task). Summative assessment is performed at the end of the semester through a written test and oral examination.

To grade students, a point rating system is used. In the semester a student may earn:

1. up to 20 credit points for in-class work;
2. up to 10 points for the computational assignment;
3. up to 10 points for the written test;
4. some additional points for conference or seminar participation.

The points are summed up and preliminary grades are offered. Preliminary grading criteria:

1. Excellent (A) – total score at least 25 points, at least 6 points for the test, and 10 points for the computational assignment;
2. Good (B) – total score at least 20 and 10 for the computational assignment;
3. Satisfactory (C) – total score at least 15, or total score ≥ 11 and ≥ 1 for in-class work;
4. Unsatisfactory (F) – otherwise.

If a student disagrees with his/her preliminary grade, he/she goes to exam. Exam grading criteria:

1. Excellent (A) – student gives full and correct answers to the examiner's questions, solves additional tasks, uses terminology correctly, demonstrates detailed knowledge of the course's

structure and logic;

2. Good (B) – student gives correct answers to most of the questions, faces some difficulties in solving additional tasks, uses terminology correctly, demonstrates good knowledge of the course's structure and logic;
3. Satisfactory (C) – student fails to answer some important questions or cannot solve additional task, understands the course's structure and logic within a specific domain but finds it difficult to point out the connection between different course's parts, the usage of terminology is mostly correct;
4. Unsatisfactory (F) – student fails to answer questions concerning basic terms of the course.

Basic Literature

1. Wolsey L.A. *Integer Programming*, John Wiley & Sons, New York (1998).
2. Garey M., Johnson D.S. *Computers and Intractability: A Guide to the Theory of NP-completeness*, W.H. Freeman and Co., San Francisco (1979).
3. Hu T.C. *Integer Programming and Network Flows*, Addison-Wesley, (1969).
4. Ford L.R., Fulkerson D.R. *Flows in Networks*, Princeton University Press (1962)

M.1-V VARIABLE PART (elective courses)

Title of the course:

M.1-V-1 Monte Carlo methods (basic course)

(course of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods")

Information about the author:

Voytishek Anton Vaclavovich

- Full Professor, Doctor of physical and mathematical sciences;
- professor of CCM DMM NSU; see http://mmfd.nsu.ru/mmf/kaf/cm/prep.asp;
- leading researcher of LSP ICM&MG SB RAS; see http://osmf.sscc.ru/mixa/bak.html;
- see also the section "Information about the supervisor of the programme" of this programme;
- e-mail: vav@osmf.sscc.ru

Course description

The course M.2-B-1 "Monte Carlo Methods (Basic Course)" (together with the course M.2-B-2 "Monte Carlo Methods (Professional Course)") is a fundamental discipline for all section M.2 "Professional cycle" of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods" and intends for studying at the first semester of the MEP. When developing the course the possibility for control of various preparation levels of Russian and foreign students in the fields of probability theory, mathematical and functional analysis, programming and numerical statistical modelling and simulation is provided.

The course presents detailed descriptions for approaches to computer realization of standard random numbers, for algorithms of numerical modelling of random variables and vectors, for methods of construction of Monte Carlo estimates (estimators) for numerical integration (including the approximation of special indefinite sums of integrals). The introduction and final lectures of the course include presentations of modern actual applied problems which imply the using of corresponding effective methods of numerical statistical modelling and simulation. The extensive seminar and independent student's work, together with the fulfillment of substantial semester task, are also planned within the course.

The *aim* of the course "Monte Carlo Methods (Professional Course)" is to acquaint student with bases of the theory of Monte Carlo methods.

The corresponding *objectives* of the course "Monte Carlo Methods (Professional Course)" are:

- to give to students basic knowledge in the theory and applications of methods for numerical statistical modelling and simulation;
- to teach students to analyze the algorithms of the Monte Carlo method in terms of their efficiency and possibility of their use in numerical solution of actual applied problems;
- to train students to construct the economical numerical algorithms for modelling of random variables (one-dimensional or multi-dimensional) and to use technologies of optimization of Monte Carlo numerical schemes.

When developing the course, the vast experience of the author in teaching at Mechanical-Mathematical and Physical Departments of NRU NSU the disciplines concerned the theory of statistical modelling and simulation, together with various author's teaching aids, are used.

The course "Monte Carlo Methods (Basic Course)" can be used in realization of the Joint Master Programme "Applied Mathematics and Energy Strategies" (MINES Paris Tech, France – NRU NSU, Russia) and international master programmes "Probability and Statistics", "Mathematical and Computer Modelling in Mechanics" and "Modern Trends in Discrete Mathematics and Combinatorial Optimization" of DMM NRU NSU.

Learning outcomes of the course

As the result of study of the course “Monte Carlo Methods (Basic Course)” the student

should know the basics of theory and applications of methods for numerical statistical modelling and simulation;

should be able to analyze the algorithms of the Monte Carlo method in terms of their efficiency and possibility of their use in numerical solution of actual applied problems;

should possess the technologies of economical numerical modelling of random variables (one-dimensional or multi-dimensional) and variance reduction for standard Monte Carlo estimates (estimators).

Course content

1. Main ideas of realization for realization of standard random number generators
2. Standard method for modelling of discrete random variables, cost
3. Special methods for modelling of discrete random variables. The quantile method
4. Standard method for modelling of continuous random variables, elementary densities
5. Modelling of random vectors
6. The integral and discrete superposition methods
7. The majorant rejection method
8. Special methods for modelling of continuous random variables. Modelling of the gamma-distribu-tion
9. Modelling of the beta-distribution
10. Modelling of isotropic vector. Numerical modelling of Gaussian random vector
11. Unbiased estimate (estimator) of mathematical expectation. Error of the standard Monte Carlo method. Expenditures and cost of the Monte Carlo method
12. Stochastic estimates (estimators) for multiple integrals. The importance sampling principle
13. Singularity inclusion into the density. Delta-densities
14. The conditional mathematical expectation method
15. The splitting method
16. Methods for variance reduction for stochastic estimate of an integral (main ideas): the main part separation method, the integration on domain part method, the group sampling
17. The simplest mathematical model of particle transfer
18. Applied Markov chains. Fredholm integral equation of the second kind for the summarized collision density function
19. Linear functionals on solution of integral equation as an indefinite sum of integrals
20. Main (“collision”) estimate for the linear functional on solution of integral equation
21. Local estimates (estimators)

Method of assessment

In the program of the course, the carrying out a colloquium and an examination is provided. Students also are offered to carry out a semester home task. The detailed grades for the home task, the colloquium and the examination are presented in the working program of the discipline **M.2-B-1**.

Basic literature

Voytishek A.V. Foundations of the Monte Carlo Method. Novosibirsk, 2013 (electronic version).

Title of the course:

M.1-V-3 Number Theory

Information about the authors:

Vdovin Evgenii Petrovich

- Associate professor, Doctor of physical and mathematical sciences;
- professor of Chair of algebra and mathematical logic (CAML) in DMM NSU; see http://mmfd.nsu.ru/mmf/kaf/aml/aml-e.html;
- deputy director of IM SB RAS; see http://math.nsc.ru/str/dir.htm;
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- Doctor of physical and mathematical sciences;
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Course description

The course contains a series of classical topics in number theory which can be informally divided into three parts: algebraic and transcendental numbers, distribution of primes, and p-adic numbers and their applications. The first section is partially motivated by the famous problem of "squaring the circle" known since the ancient times. Being unsolvable by means of straightedge and compass, this problem had been inspiring the development of algebra and geometry for centuries until it was proved in 1882 that the number π is transcendental. The course covers the definitions and basic properties of algebraic complex numbers and algebraic integers, the Liouville's theory of Diophantine approximations, and complete proofs of the theorems by C. Hermite and F. Lindemann that the fundamental constants e and π are transcendental.

In the second section, we study the foundations of the analytic number theory. Founded by P. Dirichlet and B. Riemann in the middle of XIX century, this theory remains one of the most important branches of contemporary mathematics. The main idea of the analytic number theory is to apply well-developed techniques of advanced complex analysis to the arithmetic of natural numbers. The course focuses on two famous problems of this kind: determine the asymptotic behavior of the ratio of primes in the series of natural numbers and prove that each arithmetic progression of the form $a_n = a + nd$ with relatively prime a and d contains infinitely many primes. Solutions of these problems are given by the famous Prime Number Theorem and the Dirichlet Theorem, respectively. The course contains complete proofs of these statements based on the properties of the Riemann zeta-function, Dirichlet series, and Chebyshev functions.

The third section contains the basics of p-adic number theory which plays an important role in contemporary geometry and mathematical physics, as well as in the general theory of Diophantine equations. We observe the theory of fields equipped with a valuation, classify the valuations on the field of rational numbers, and state the construction of the field of p-adic numbers. Finally, we establish the principal properties of p-adic numbers and shortly review some of their applications.

Prerequisites assumed to the students taking the course include: elementary number theory (division algorithm, fundamental theorem of arithmetic, Euclidean algorithm), basic algebra (ring of polynomials, structure of finite abelian groups), advanced complex-valued calculus (convergence of functional series and of parametrized improper integrals, Cauchy integral theorem), and basic topology (definition of a topological space, convergence, and topological complements).

Course content

1. The field of algebraic numbers and the ring of algebraic integers
2. Diophantine approximation of a real number by rational numbers. The Dirichlet Theorem on rational approximation. An example of a transcendental number
3. The numbers e and π are transcendental
4. The distribution of primes: statement and history. Chebyshev functions
5. Discrete convolution and the product of Dirichlet series for arithmetic functions. Mobius inversion formula
6. Euler identity. Riemann zeta-function in the half-plane Re(z)>1
7. The connection between the integral Chebyshev function and the logarithmic derivation of the Riemann function
8. The analytic extension for the Riemann function in the half-plane Re(z)>0. Zeros of the Riemann function in Re(z)≥1
9. Proof of the asymptotic law for the distribution of primes. An asymptotic formula for the n^{th} prime
10. Group of characters for a finite abelian group. Orthogonality relations
11. Dirichlet theorem for the number of primes in an arithmetic progression
12. Valuation fields. The classification of valuations on the field of rational numbers. Complement of a valuation field
13. The construction and properties of the ring of integer p-adic numbers and of the field of p-adic numbers

Method of assessment

In the program of the course, the carrying out the examination is provided. The detailed grade for this examination is presented in the working program of the discipline.

Basic literature

Vdovin E.P., Kolesnikov P.S. Number Theory. Novosibirsk, 2013 (electronic version).

Title of the course:

M.1-V-4 History of numerical statistical modelling and simulation

(course of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods")

Information about the author:

Voytishek Anton Vaclavovich

- Full Professor, Doctor of physical and mathematical sciences;
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Course description

The role of numerical statistical modelling and simulation (or Monte Carlo methods) increases with the development of computer technologies. Historically, the development of Monte Carlo methods was related to progress in mathematical modelling of nuclear processes (in order to produce corres-ponding technologies) in USSR and USA in fifties of the 20-th century.

Development beginning of the theory and applications of the Monte Carlo algorithms is connected with fundamental works of J.Neumann, S.Ulam, N.Metropolis, N.I.Buslenko, J.M.Hammmersley, J.Spanier, I.M.Sobol, S.M.Ermakov, G.A.Mikhailov, G.I.Marchuk, M.Kalos and members of their scientific schools.

Over the past half-century the applicability for methods of numerical statistical modelling and simulation greatly expended. The theory of probability representations for solutions of problems of mathematical physics was elaborated. On basis of this theory, the effective Monte Carlo estimates were constructed. Effective numerical algorithms were also elaborated for statistical physics (the Metropolis-Hastings algorithm, the Ising model), physical and chemical kinetics (multi-particle problems, solution of Boltzmann and Smoluchowski equations, modelling of reactions and phase transitions), queueing theory, financial mathematics, turbulence theory, mathematical biology, etc. The Monte Carlo algorithms allow effective parallelization for calculations on modern supercomputer equipment. In development of the mentioned above theories and numerical schemes, the significant (sometimes leading) role belongs to researches from Department of Statistical Modelling in Physics of Institute of Computational Mathematics and Mathematical Geophysics of Siberian Division of Russian Academy of Sciences (head – Corresponding Member of USSR Academy of Sciences, Professor G.A.Mikhailov).

In the course “History of numerical statistical modelling and simulation” main stages of development of the theory and applications of Monte Carlo numerical algorithms are reflected. Special place in this course takes the review of scientific achievements of Novosibirsk school of Monte Carlo methods.

Learning outcomes of the course

As a result of study of the course “History of numerical statistical modelling and simulation” the student acquires detailed knowledge on development of theory and applications of Monte Carlo methods. It allows him to choose more competently his training trajectory in frames of the MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods” and also the further scientific and expert activity in this field.

Course content

1. Buffon’s needle experiment
2. E.Fermi’s Monte Carlo experiments in the field of neutron diffusion.
3. Monte Carlo experiments in the Los Alamos Scientific Laboratory. Manhattan Project.
4. Review of works of Von Neumann, S.Ulam, N.Metropolis in the field of Monte Carlo methods
5. Review of the paper *Metropolis N., Ulam S. The Monte Carlo method // Journal of American Statistical Association. 1949. V. 44, № 249. P. 335-341*.
6. The nuclear project in the Soviet Union
7. Review of early works of G.I.Marchuk, G.A.Mikhailov, S.M.Ermakov, I.M.Sobol, N.N.Chentsov, D.A.Frank-Kamenetskii, N.S.Bahvalov, etc. in the field of Monte Carlo methods
8. Review of the monography *Buslenko N.P., Golenko D.I., Sobol I.M., Sragovich, V.G., Shreider Ju.A. Method of Statistical Trials (Monte Carlo Method). Moscow: State Publishing House of Physical-Methematical Literature, 1962 [in Russian]*.
9. Review of the monorgaphy *Hammmersley J.M, Handscomb D.C. Monte Carlo Methods. New York: Jonh Wiley and Sons, 1964.*
10. Review of the monorgaphy *Spanier J., Gelbard E. Monte Carlo Principles and Newtron Transport Problems. Addison-Wesley, Reading, 1969.*
11. On Moscow Monte Carlo scientific school. Review of the monorgaphy *Sobol I.M. Numerical Monte Carlo Methods. Moscow: Nauka, 1973 [in Russian]*.
12. On Leningrad (St-Petersburg) Monte Carlo scientific school. Review of the monorgaphy Ermakov

S.M. Monte Carlo Method and Related Issues. Moscow: Nauka, 1974 [in Russian]

13. On Novosibirsk Monte Carlo scientific school. Review of the monorgaphy *Marchuk G.I., Mikhailov G.A., Nazaraliev M.A., Darbinjan R.A., Kargin B.A., Elepov B.S. The Monte Carlo Methods in Atmospheric Optics. Heidelberg: Springer-Verlag, 1980.*

14. Prof. G.A.Mikhailov as the Novosibirsk Monte Carlo scientific school.

15. The outstanding role of the textbook *Ermakov S.M., Mikhailov G.A. Statistical Modelling. Moscow: Nauka, 1982 [in Russian]* for development of theory and applications of Monte Carlo methods.

16. Blossoming of the Novosibirsk Monte Carlo scientific school – eightieth years of the twentieth century. Review of the monorgaphy. Mikhailov G.A. Optimization of Weighted Monte Carlo Methods. Heidelberg: Springer-Verlag, 1992.

17. Survival of the Novosibirsk Monte Carlo scientific school – ninetieth years of the twentieth century.

18. Revival of the Novosibirsk Monte Carlo scientific school in 2000-2013.

19. Monte Carlo methods teaching in Novosibirsk State University.

20. Review of the textbook *Mikhailov G.A., Voytishek A.V. Numerical Stochastic Modelling. Monte Carlo Methods. Moscow: Academia, 2006 [in Russian]*.

Method of assessment

In the program of the course, the carrying out the test is provided.

Basic literature

Voytishek A.V. Foundations of the Monte Carlo Method. Novosibirsk, 2013 (electronic version).

Title of the course:

M.1-V-5 Inverse problems

(course of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods")

Information about the author:

Kabanikhin Sergey Igorevich

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- head of the Laboratory of mathematical problems of geophysics of ICM&MG SB RAS; see http://www.sscc.ru/index_e.html;
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Course description

The *aim* of the course "Inverse problems" is to acquaint students with actual theoretical and applied aspects of numerical algorithms related to numerical approximation of unknown parameters of mathematical models.

The corresponding *objectives* of the course "Inverse problems" are:

- to give to students basic knowledge in the modern theory of inverse problems and its applications;
- to teach students to analyze the modern difference numerical schemes for solution of inverse problem in terms of their correction, convergence degree and conditional stability.

When developing the course, the vast experience of the author in teaching at

Mechanical-Mathematical Department of NRU NSU the disciplines related to the modern theory of inverse problems, together with author's fundamental teaching aids and recent scientific articles, are used.

The course "Inverse problems" can be used in realization of the Joint Master Programme "Applied Mathematics and Energy Strategies" (MINES Paris Tech, France – NRU NSU, Russia) and international master programmes "Probability and Statistics", "Mathematical and Computer Modelling in Mechanics" and "Modern Trends in Discrete Mathematics and Combinatorial Optimization" of DMM NRU NSU.

Learning outcomes of the course

As the result of study of the course "Inverse problems" the student
should know basic sections of modern theory of inverse problems;
should be able to analyze the modern algorithms for numerical solution of inverse problems in terms of their correction, convergence degree and conditional stability.

Course content

(see http://mmf.nsu.ru/sites/default/files/InvProblems-Kabanikhin-2012.doc [in Russian])

1. Correct, incorrect and conditionally correct problems. Correctness set. Conditional stability.
2. Examples of incorrect problems: differentiation, Cauchy problem for the Laplace equation, problem for the thermal conductivity equation with inverse time, integral equations of the first kind, operator equations $Aq=f$ with a compact operator A, tomography.
3. Regularization methods of A.N.Tikhonov and M.M.Lavrentyev. Discrete and iterative regularization.
4. Theorem on continuous of the inverse operator. Quasi-solution. V.K.Ivanov theorem.
5. Theorem on singular decomposition of matrixes. S.K.Godunov method for regularization of systems of linear algebraic equations.
6. Compact operators in separable Hilbert spaces and their properties. Singular decomposition of a compact operator. Operator equation of the first kind: $Aq=f$. C.E.Picard criteria for solvability.
7. Frechet derivative. Goal functional $J(q)=<Aq-f,Aq-f>$ and its gradient $J'q$. Ratio for the gradient $J'q=2[A'q](Aq-f)$.
8. Uniqueness conditions for stationary point of the goal functional.
9. Direct and inverse problem for the string fluctuation equation. Theorems of existence and uniqueness of the solution of direct and inverse problems for the string fluctuation equation. Calculation of gradient of goal functional for the inverse string fluctuation equation. Method of the difference scheme inversion.
10. Inverse acoustic problem and methods of its solution.
11. Convergence with respect to the functional of the common iteration method and the gradient descent method. Evaluation of the convergence with respect to the functional degree for the common iteration method and the gradient descent method.
12. Continuous module of inverse operator and evaluation of conditional stability.
13. Cauchy problem for the thermal conductivity equation with inverse time. Evaluation of conditional stability. Cauchy problem for the Laplace equation. Evaluation of conditional stability.
14. Strong convergence for the gradient descent method.
15. Linearization method. Newton-Kantorovich method. Gelfand-Levitan-Krein method.

Method of assessment

In the program of the course, the carrying out the test and the examination are provided.

Basic literature

Kabanikhin S.I. Inverse and Ill-posed Problems. De Gruyter, Germany, 2011.

Title of the course:

M.1-V-6 Combinatorial Optimization

(course of the MEP "Modern Trends in Discrete Mathematics and Combinatorial Optimization")

Information about the author:

Kononov Alexander Veniaminovich

- Associated professor of the Chair of Theoretical Cybernetic in Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU);
- Senior researcher of Laboratory of Mathematical Models of Decision making of Sobolev Institute of Mathematics of Siberian Branch (SB) of RAS; see http://www.math.nsc.ru/LBRT/k5/lab.html
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Course description

There are many reasons for studying algorithms. The primary one is to enable students to use computer efficiently. A novice programmer, without basic knowledge of algorithms, may prove to be a disaster to his company where he works. The study of algorithms is the main objective of this course.

One of the most important problems of modern society is the processing of vast amounts of data to make the best decision. One of the promises of the information technology era is that many decisions can now be made rapidly by computers. The science of how to make decisions in order to achieve some best possible goal has created the field of combinatorial or discrete optimization.

Combinatorial optimization is one of the most active areas of discrete mathematics. It has been the subject of extensive research for over fifty years. Combinatorial optimization has its roots in combinatorics, theoretical computer science and operations research. A main motivation is that hundreds or thousands of real-life problems and phenomena can be considered as combinatorial optimization problems. Therefore, the design of combinatorial algorithms now is a vast area containing many strong techniques for various applied problems.

The course «Combinatorial Optimization» includes the fundamentals of graph theory, linear and integer programming, and complexity theory. The course covers classical topics in combinatorial optimization as well as very recent results. It consists of two big parts. Most of the problems considered in the first part have efficient algorithms. Optimal solutions of these problems can be found in time polynomial in the size of the input. While most of the problems studied in the second part of the course are NP-hard. A polynomial-time algorithm for NP-hard problems is unlikely to exist. However, in many cases one can at least find approximation algorithms with provably good performance.

The course «Combinatorial Optimization» belongs to the base part of the professional cycle of MEP «Modern trends in discrete mathematics and combinatorial optimization» since it provides theoretical knowledge that is useful for studying the other courses of the MEP.

Learning outcomes of the course

As the result of study of the course «Combinatorial Optimization» the student

should know

1. the typical discrete optimization problem, the combinatorial algorithms, and the formulations of the most important theorems,
2. classes P and NP, the notions NP-completeness and NP-hardness;
3. proof ideas of the theorems;
4. methods and techniques of combinatorial optimization including greedy algorithms, dynamic programming, linear programming, matroids, flow algorithms, approximation algorithms and schemes;

should be able

5. to determine the combinatorial complexity of new combinatorial problems;
6. to apply the methods of combinatorial optimization for solving problems of discrete mathematics;
7. to reduce new combinatorial problems to the problem studied in the course;

should possess the various techniques and methods for analyzing and solving combinatorial optimization problems.

Course content

1. Graphs: connectivity criteria, characterization of trees, bipartite graphs, Euler's graphs. (4 hours)
2. Introduction in linear programming and integer linear programming. (2 hours)
3. Spanning trees and Arborescences: Kruskal's algorithm, Prim's algorithm, Edmonds' branching algorithm. (2 hours)
4. Shortest Paths: Dijkstra's algorithm, Moore-Bellman-Ford algorithm, Floyd-Warshall algorithm. (2 hours)
5. Network Flows: Ford-Fulkerson algorithm, Menger's theorems, Edmonds-Karp algorithm, Goldberg-Tarjan algorithm. (4 hours)
6. Matching problems: Tutte matrix and Lovász randomized algorithm, Berge-Tutte theorem, Edmonds' Cardinality Matching algorithm. (6 hours)
7. Matroids: characterization of matroids and Edmonds-Rado theorem. (2 hours)
8. Introduction in complexity theory. (classes P and NP, NP-completeness, Cook theorem) (4 hours)
9. Approximation algorithms and schemes. (2 hours)
10. Knapsack and Bin-packing problem: Weighted Median algorithm, Dynamic programming and an FPTAS, Fernandes-de-la-Vega-Lueker algorithm. (2 hours)
11. Scheduling problems: Graham's algorithm, LPT rules, Novicki & Smutnicki algorithm. (2 hours)
12. The Traveling Salesman Problem: Double-Tree algorithm, Christofides-Serdyukov algorithm. (2 hours)

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. B. Korte and J. Vygen. *Combinatorial optimization.* Springer, Berlin, Germany, fourth edition, 2007.
2. M.R. Garey and D.S. Johnson. *Computers and Intractability: A Guide to the theory of NP-completeness*. W.H. Freemann and Company, San Francisco, CA, 1979.
3. D.S. Hochbaum, editor. Approximation algorithms for NP-hard problems. PWS-publishing Company, Boston, MA, USA, 1997.

Title of the course:

M.1-V-7 Stochastic models of meteorological processes

(course of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods")

Vasiliy Aleksandrovich Ogorodnikov

- Associate Professor, Doctor of physical and mathematical sciences;
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- major researcher of LSP ICM&MG SB RAS; see http://osmf.sscc.ru/mixa/bak.html;
- see also the section «Information about the author» of working program of the discipline **M.2-V-13**;
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Course description

The proposed course examines issues related to the construction of stochastic models of hydrometeorological processes, fields, and their complexes based on real observations. Approaches to the construction of these models are based on the use of simulation algorithms of Gaussian and non-Gaussian processes and fields with a given one-dimensional distributions and correlation functions. Due to the specificity and complexity of real meteorological processes (fields) which as a rule are not stationary (heterogeneous) the need arises to adapt the existing algorithms to real process or develop new special algorithms. This course considers approaches to solving these problems, as well as methods of verification of models and algorithms for the calculation of different characteristics associated with extreme weather events with the help of model samples. The issues related with stochastic interpolation of meteorological fields from stations in the grid nodes and approaches to building dynamic-stochastic models of atmospheric processes are discussed.

Course "Stochastic models of meteorological processes" will meet high international level in the field of scientific research on the theory and applications of algorithms for discrete stochastic numerical simulations. The course will also help master student to receive research skills. The course program will include a review of new research results on the use of statistical modelling for the study of real processes. This will contribute to the further development of the Master Education Programme "Numerical Statistical Modelling. Monte Carlo Methods".

The course is associated with the implementation of Development Program NRU NSU in 2009-2018 years and related to program sections "Discrete and Computational Mathematics", "Modelling and analysis of the results of physical experiments", "Technologies of distributed and high-performance computing and systems" of the direction "Mathematics, fundamental basis of computer science and information technology".

Learning outcomes of the course

After studying of the course "Stochastic models of meteorological processes" a master student will know modern methods of investigation of real processes with the help of numerical stochastic models. He will be also familiar with the modern approaches to the construction of stochastic models of real meteorological processes and be able to build stochastic models of different processes by the real data. He will know the methods of verification of models and methods for the numerical studies using models of different statistical characteristics of investigated process.

Course content

1. Principles of construction of numerical stochastic models of real meteorological processes
2. Assignment of the input data for models. Approximation of empirical probability distributions of meteorological time series
3. Stochastic models of scalar meteorological time series

4. Models of vector time series of wind speed
5. Models of joint non-Gaussian time series of several different meteorological elements
6. Periodically correlated meteorological stochastic processes
7. Simulation of spatial and spatial-temporal fields of meteorological elements
8. Specific of modelling of time series and fields of precipitation sums
9. Numerical simulation of indicators of an output of meteorological process for the given levels on the basis Markov chain
10. Study the characteristics of extreme meteorological events on the basis of numerical stochastic models
11. Numerical models of conditionally distributed meteorological fields. Stochastic interpolation of meteorological fields
12. Stochastic dynamic models of meteorological processes
13.Control questions, tests

Method of assessment

In the program of the course, the carrying out an examination is provided. The detailed grade for this examination is presented in the working program of the discipline **M.2-V-13**.

Basic literature

Ogorodnikov V.A. Stochastic Models of Meteorological Processes. Novosibirsk, 2013 (electronic version).

Title of the course:

M.1-V-8 Modern problems of numerical mathematics

(course of the MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”)

Information about the author:

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- Professor of the Chair of computational mathematics (CCM) in Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU); see http://mmfd.nsu.ru/mmf/kaf/cm/prep.asp;
- major researcher of Laboratory of numerical analysis and computer graphics of Institute of Computational Mathematics and Mathematical Geophysics (ICM&MG) of Siberian Branch (SB) of RAS; see http://www.sscc.ru/index_e.html (section “Members of RAS”);
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Course description

The course “Modern problems of numerical mathematics” contains basic information on numerical methods of linear algebra, data about modern difference schemes of numerical mathematics and review of actual applications of numerical mathematics. It is also includes seminars for training in solving test and advanced problems of numerical mathematics.

The *aim* of the course “Modern problems of numerical mathematics” is to acquaint students with actual theoretical and applied aspects of numerical mathematics.

The corresponding *objectives* of the course “Modern problems of numerical mathematics” are:

- to give to students basic knowledge in the theory and applications of modern numerical mathematics;

- to teach students to analyze the modern difference numerical schemes in terms of their error, stability, conservatism and possibility of their use in numerical solution of actual applied problems;
- to train students to solve test and advanced problems of numerical mathematics.

When developing the course, the vast experience of the author in teaching at Mechanical-Mathematical Department of NRU NSU the disciplines related to the modern problems of numerical mathematics, together with author's fundamental teaching aids and recent scientific articles, are used.

The course "Modern problems of numerical mathematics" can be used in realization of the Joint Master Programme "Applied Mathematics and Energy Strategies" (MINES Paris Tech, France – NRU NSU, Russia) and international master programmes "Probability and Statistics", "Mathematical and Computer Modelling in Mechanics" and "Modern Trends in Discrete Mathematics and Combinatorial Optimization" of DMM NRU NSU. The course corresponds to the obligatory discipline "Modern problems of applied mathematics and information science" of the State Master Standard (SMS) for training direction 010400 – "Applied Mathematics and Information Science" (see the site http://fgosvpo.ru/uploadfiles/fgos/30/20110325143339.pdf [in Russian]).

Learning outcomes of the course

As the result of study of the course "Modern problems of numerical mathematics" the student

should know basic theoretical and applied problems and schemes of modern numerical mathematics;

should be able to analyze the modern algorithms of numerical mathematics in terms of their convergence, stability and conservatism;

should possess the methods for solving test and actual problems of the modern numerical mathematics.

Course content

1. Linear operator equation in a finite-dimensional Hilbert space. Mathematical models of the conjugate factorized structure. Error functional. Generalized solution. Spectral condition number.
2. Double-layer iterative methods. Relevant and sufficient condition of convergence. Optimal double-layer stationary method. Preconditioner. Alternately triangular preconditioner. Non-stationary double-layer method with the Chebyshev set of iterative parameters.
3. Iterative methods of the variational type: the gradient descent method, minimal residual method. Asymptotic property, resonant mode. Determination of the spectrum boundaries. Optimal adaptive alternately triangular preconditioner.
4. Optimal adaptive three-layer Chebyshev iterative method. Optimal adaptive method of conjugate gradients.

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

Konovalov A.N. Introduction to Numerical Methods of Linear Algebra. Novosibirsk: Nauka, 1993 [in Russian]

M.2 PROFESSIONAL CYCLE, THE MAIN (VARIABLE) PART
M.2-C CORE COURSES

Title of the course:
M.2-C-1 Advanced Probability

Information about the author:
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Course description

The applications of Probability Theory to many areas in Mathematics and in other fields have multiplied dramatically in recent years. Rich interactions with classical analysis have been found in the study of random fractals; researchers in Combinatorics, Computer Sciense, Machine Learning, high-dimensional Probability and, of course, Statistics increasingly need sophisticated probabilistic tools. The aim of this course is to provide such tools, while also preparing the students for more advanced courses in Stochastic Analysis and Statistics.

The course "Advanced probability" is designed to prepare professionals with deep knowledge of probability theory and the skills to use this knowledge to further research. The course covers the main sections of the modern theory of summation of independent random variables.
The *main goal* of the course "Advanced probability" is to acquaint students with the techniques prove the main limit theorems of modern probability theory.

The corresponding *objectives* of the course Advanced probability" are:

1. to introduce students to the basic concepts and methods of the modern theory of summation of independent random variables;
2. to give the skills to use this knowledge in further research and practical work;
3. to give an idea of the current state and development of the area;
4. to train students to solve test and advanced problems of probability theory.

The main focus will be on probability inequalities for sums of independent random variables and strong laws. After a brief introduction to Probability Theory, the focus will be on issues of immediate use to statisticians, such as modes of convergence, limit theorems, laws of large numbers, and other topics.

Learning outcomes of the course

As the result of study of the course "Advanced probability" the student
should know basic concepts and methods of the modern theory of summation of independent random variables;
should be able to use this knowledge in further research and practical work;
should possess the methods for solving test and advanced problems of probability theory.

Course content

The course "Advanced probability" consists of three connected parts: Basic concepts of Probability Theory; Probability inequalities; Laws of large numbers and the Law of the iterated logarithm.

The first part of this course presents basic concepts of probability theory, basic inequalities and limit theorems.

Some of the most useful tools in probability theory and mathematics for proving finiteness or convergence of sums and integrals are inequalities. There exist many useful ones spread out in books and papers. In the second part of the course we make an attempt to present a sizable amount of the most important inequalities. The second part will explore probability inequalities for sums of independent random variables: exponential inequalities, inequalities for the distribution of the maximum of independent random variables, symmetrization inequalities.
In particular, we will study in detail the Nagaev - Fuc inequality, which, in contrast to the classical probability inequalities, not assumed the existence of the finite moments of the random variables of certain orders.

We also will consider in detail the moment inequalities - Rosenthal inequality and its consequences.

The last part of the course is devoted to the investigation of the convergence of series of independent random variables and laws of large numbers. The law of large numbers states that the distribution of the arithmetic mean of a sequence of independent trials stabilizes around the center of gravity of the underlying distribution (under suitable conditions). There exist *weak* and *strong* laws and several variations and extensions of them. We shall meet some of them as well as some applications. In particular, we will prove the Marcinkiewicz - Zygmund law of large numbers. We also study the rate of convergence in the law of large numbers.

In the third part of the course we also give a detailed proof of the Hartman – Wintner law of the iterated logarithm. This is a special, rather delicate and technical, and very beautiful, result, which provides precise bounds on the oscillations of sums of the above kind. The name obviously stems from the iterated logarithm that appears in the expression of the parabolic bound.

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. *Borovkov A. A.* Probability Theory. New York: Gordon & Breach, 1998.
2. *Lamperti, J. W.* Probability. A survey of the mathematical theory. 2nd ed. New York, NY: Wiley, 1996.
3. *Petrov V. V.* Sums of independent random variables. Berlin: Akademie-Verlag, 1975.
4. *Petrov V. V.* Limit theorems for the sums of independent random variables. New York: Springer-Verlag, 1975.
5. *Shiryaev A. N.* Probability. New York, NY: Springer-Verlag, 1995.
6. *Shiryaev A. N.* Problems in probability. New York, NY: Springer, 2012.
7. *Bauer H.* Wahrscheinlichkeitstheorie. de Gruyter, Berlin, 1991.
8. *Gut A.* Probability: A Graduate Course. Springer-Verlag, New York, 2005.
9. *Kallenberg O.* Foundations of Modern Probability. Springer-Verlag, New York, 1997.

Title of the course:
M.2-C-2 Martingale Theory

Information about the author

Baklanov Evgeny Anatol'evich
- Candidate (PhD) of physical and mathematical sciences;
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Course description

Theory of Stochastic Processes plays an important role in Probability Theory and its applications in Statistics, Mathematical Finance, Functional Analysis, etc. Martingale Theory, in particular, plays a crucial role in the development of stochastic integration as the main tool for solving stochastic differential equations.

The aim of this course is to give the students a solid introduction to the basic theory of stochastic processes in discrete and in continuous time with special focus on martingales. The course "Martingale Theory" is designed to prepare professionals with deep knowledge of probability theory and stochastic processes and the skills to use this knowledge to further research. The course covers the main sections of the modern martingale theory and stochastic integration.

The *main goal* of the course "Martingale Theory" is to develop students' skills in the use of probabilistic methods in the study of sequences of dependent random variables - martingales.

The corresponding *objectives* of the course "Martingale Theory" are:

- to introduce students to the basic concepts and methods of modern martingale theory and stochastic integration;
- to give a solid understanding of the theory of martingales and its applications e.g. to mathematical finance;
- to train students to solve test and advanced problems of martingale theory and stochastic integration.

The main focus will be on fundamental inequalities and convergence theorems for martingales with discrete and continuous time, and uniformly integrable martingales.

Learning outcomes of the course

As the result of study of the course "Martingale Theory" the student
should know basic concepts and methods of the modern martingale theory and stochastic integration;
should be able to use this knowledge in further research and practical work;
should possess the methods for solving test and advanced problems of martingale theory and stochastic integration.

Course content

The course "Martingale Theory" consists of three connected parts: Martingale Theory; Gaussian Processes; Stochastic Integration.

1. Martingale Theory

Conditional expectations: definition, existence and uniqueness, basic properties.
Martingales: discrete and continuous time. Martingales, submartingales, supermartingales. Definitions and basic properties. Examples.
Doob decomposition. Stopping times. Upcrossings. Basic inequalities.
Convergence theorems. Uniformly integrable martingales.

2. Gaussian processes

Brownian motion (Wiener process): definition and basic properties.
The construction of the continuous Brownian motion. Path properties of Brownian motion.
The law of the iterated logarithm for Brownian motion.
The quadratic variation of Brownian motion. The distribution of the maximum of Brownian motion.

3. Stochastic Integration

Ito integral for simple processes. Ito isometry. Constructing Ito integral for general square integrable processes. Ito integral. Existence and properties. Ito process.
Ito's change of variables formula. Multidimensional Ito formula.
Continuity and martingale properties of Ito process.
Stochastic differential equation. Weak and strong solutions.

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. Shiryaev A. N. *Probability.* New York, NY: Springer-Verlag, 1995.
2. Oksendal B. *Stochastic Differential Equations*, New York, NY: Springer-Verlag, 2003.
3. Brzeźniak Z., Zastawniak T. J. *Basic Stochastic Processes: A Course Through Exercises*. Springer, 1999.
4. Kallenberg, O. *Foundations of Modern Probability.* Springer-Verlag, New York, 1997.
5. Steele M. *Stochastic Calculus and Financial Applications.* Springer, 2001.

Title of the course:

M.2-C-3 Random Walks

Information about the author:

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Course description

Original lecture course Random Walks was developed for teaching foreign students studying Probability and Statistics in English. This is a basic course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics” , “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability Theory and Stochastic Processes and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - “Mathematics”. As a prerequisite, a modest understanding of Probability theory at an undergraduate level and of some basic notions in Stochastic Processes is assumed.

We present various techniques for proving limit theorems for random walks, such as method of characteristic functions, method of moments and Stein's method.We prove both integral and local limit theorems, and key renewal theorem as well. Connections with applications from queueing theory and risk theory are discussed. The large deviation theory for the supremum of a random walk is presented as well, for both light-tailed and heavy-tailed distributions.

The aim of the course “Random Walks” is to acquaint students with actual theoretical aspects of random walks theory, to provide student with the knowledge and ability to apply modern methods of research in the analysis of random walks arising in applications.

The *objectives* of the course “Random Walks” are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics.

Learning outcomes of the course

As the result of study of the course “Random Walks” the student
should understand the role and place of this discipline among the other mathematical subjects;
should know the definitions, basic properties and examples of random walks and their practical value;
should be able to apply their knowledge to solve mathematical problems.

Course content

Ch a p t e r I. **Limit theorems for random walks**

1. Some basic results from Probability Theory
2. Examples of random walks
3. Single server queueing system
4. Ruin Probabilities in Cramer-Lundberg Model
5. Characteristic functions and their properties
6. Inversion formulas for characteristic functions
7. Theorem of continuity for characteristic functions
8. Central limit theorem
9. Local limit theorem in lattice case
10. Local limit theorem for densities
11. Local limit theorem for intervals
12. On the recurrence of integer valued random walk

Ch a p t e r II. **Renewal theory**

13. Denitions
14. Law of large numbers for the renewal process
15. Central limit theorem for the renewal process

16. Integral renewal theorem
17. Key renewal theorem
18. Renewal theory on the whole real line

Ch a p t e r III. **Subexponential distributions**

19. Heavy-tailed and light-tailed distributions
20. Long-tailed functions and their properties
21. Long-tailed distributions
22. Long-tailed distributions and integrated tails
23. Subexponential distributions on the positive half-line
24. Subexponential distributions on the whole real line
25. Subexponentiality and weak tail-equivalence
26. Strong subexponential distributions
27. Kesten's bound
28. Subexponentiality and randomly stopped sums

Ch a p t e r IV. **Ladder structure of random walk**

29. Denition of ladder heights and epochs
30. Taboo renewal measures
31. Asymptotics for the first ascending ladder height

Ch a p t e r V. **Maximum of random walk**

32. Asymptotics for the maximum of a random walk with a negative drift in heavy-tailed case
33. Cramer-Lundberg approximation in light-tailed case
34. Explicitly calculable ascending ladder heights
35. Single server queueing system
36. Ruin probabilities in Cramer-Lundberg model

Method of assessment

Oral exam.

Literature

1. Asmussen, S.: Applied Probability and Queues, 2^{nd} Edn. Springer, New York (2003).
2. Asmussen, S.: Ruin Probabilities. World Scientific, Singapore (2000).
3. Feller, W.: An Introduction to Probability Theory and Its Applications, vol. 2. Wiley, New York (1971).
4. Foss, S., Korshunov, D., Zachary, S.: An Introduction to Heavy-Tailed and Subexponential Distributions. Springer (2013).
5. Korshunov, D.: Random Walks. Novosibirsk, NSU (2013, electronic version).
6. Rolski, T., Schmidli, H., Schmidt, V., Teugels, J.: Stochastic Processes for Insurance and Finance. Wiley, Chichester (1998).

M.2-E ELECTIVE COURSES

Title of the course:

M.2-E-1 Applied Statistics

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Course description

Statistics is the mathematical study of the collection, organization, analysis, interpretation and presentation of randomness and variability of data. It is one of the few major disciplines in which the researcher's art and experience can have significant impact in such different fields as as biological sciences and medicine, finance and insurance, telecommunication and manufacturing, geosciences and climatology, as well as economics and education.

The course “Applied Statistics” is an elective course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics” , “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course requires the undergraduate courses on Probability Theory and Mathematical Statistics as prerequisites and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - “Mathematics”.

The aim of the course is to allow students to deepen their statistical analysis knowledge, to provide their ability to solve practical problems in various work contexts. This course will provide an introduction to modern applied statistical methods. The course will cover, in particular, the following topics: methods of sampling data and experimental design, non-parametric statistical methods, correlation analysis, simple and multiple regression, principal component analysis, one-way and two-way analysis of variance, time series analysis.

The *objectives* of the course “Applied Statistics” are to provide students with a strong foundation in mathematical and statistical methodology, experience in its applications, a solid background in the use of statistical methods, and the skills to communicate the results of statistical analysis.

Learning outcomes of the course

As the result of studying the course “Applied Statistics” the student
should understand the role and place of statistical methods in applications;
should understand the basic statistical inference methods;
should be able to understand fundamental concepts in statistical analysis;

should be able to apply their knowledge to solve practical problems requiring the use of statistical methods and to choose appropriate and adequate ones.

Course content

1. Statistical background. Sampling distributions. Point and interval estimation. Basic statistical hypotheses, tests of parametric hypotheses, goodness-of-fit tests.

2. Regression analysis. Regression models and the least squares estimation. Simple and multiple linear regression. Regression diagnostics. Confidence intervals and tests of parameters. Variable selection.

3. One-way and two-way analysis of variance.

4. Rank and order statistics and their properties. Nonparametric hypotheses testing. Wilcoxon signed-ranks test, Mann-Whitney U test, binomial sign test, Cochran Q test, Wilcoxon matched-pairs signed-ranks test, Kruskal-Wallis one-way analysis of variance, Friedman two-way analysis of variance, chi-squared goodness-of-fit test, chi-squared test for homogeneity and independence, tests of randomness, correlation tests.

6. Applied time series analysis. Regression techniques for modeling trends, smoothing techniques, autocorrelation, partial auto-correlation, moving average, Box-Jenkins models, forecasting, model selection and estimation.

Method of assessment

The final grade bases on the result of test. Course program contains sample test questions and problems.

Basic literature

1. *Encyclopedia of Statistical Sciences*, 16 Volume Set, 2nd Edition. S.Kotz (Editor-in-Chief). Wiley-Interscience, 2006, 9420 pages.

2. E. L. Lehmann, and Joseph P. Romano. *Testing statistical hypotheses*. Springer Texts in Statistics: Springer, New York, 3rd ed., 2005.

3. Norman R. Draper, Harry Smith. *Applied regression analysis*. Wiley, New-York, 3rd ed., 1998, 706 pages.

Title of the course:

M.2-E-2 Insurance Theory

Information about the author:

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Course description

The course “Insurance Theory” is an elective course in the main (variable) section of professional cycle of Master Educational Programme (MEP) “Probability and Statistics” of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability

Theory and Stochastic Processes and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - “Mathematics”. The course focuses on Insurance Risk Theory. This term is a synonym of non-life insurance mathematics. Thus, the life insurance is not discussed at all.

The course "Insurance Theory" is addressed to students interesting in application of probabilistic knowledge in finance and non-life insurance. It can be studied in parallel with a course of Financial mathematics, as they do not focus on the same issues. The course provide students with a basic understanding of mathematical methods and models of actuarial mathematics. The course consists of 36 hours of lectures and 36 hours of individual training.

The appearance of the modern risk theory goes back to the works of F.Lundberg and H.Cramèr. They realized in the first half of the 20th century that the theory of stochastic processes provides the most appropriate framework for modeling the claims arriving in an insurance business. The Poisson process was proposed by F.Lundberg in 1903 as a simple process in solving the problem of the first passage time. In 1930 H.Cramèr extended the Lundberg’s work for modeling the ruin of an insurance company as a first passage time problem. The basic model in insurance theory is called a Cramèr - Lundberg model or classical risk model. This model is the cornerstone of the mathematics of insurance. Since its introduction it was developed and generalized in different directions and, in fact, stimulated the development of research in several other areas of probability theory, such as queueing theory, extreme value theory, renewal theory, and stochastic networks.

The course will discuss the following topics of risk theory and the theory of insurance: individual and collective risk models, comparison of risks, risk insurance and reinsurance.

The *aim* of the course “Insurance Theory” is to introduce students to probabilistic approach of actuarial theory, to provide the student with the knowledge and ability to construct, research and apply the mathematical models of risk and insurance theory.

The *objectives* of the course “Insurance Theory” are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics and in financial applications.

Learning outcomes of the course

As the result of studying the course “Insurance Theory” the student

should understand the role and place of this discipline among the other mathematical subjects;

should know the practical value of Insurance Theory;

should be aware of the fundamental resources and notation within the Risk Theory and Insurance Theory;

should be able to formulate basic risk models in insurance and use it for control of probabilities of ruin.

should be able to display understanding of the submitted task and the knowledge about the theoretical background;

should be able to work independently according to the approved time schedule;

should be able to present acquired results in oral and written way.

Course content

1. **Basic risk model.** Claim number (counting) process. Risk process.
2. **Probabilistic underground.** Probability generating function and moment generating function. Characteristic function and Laplace tranform. Hazard rate function. Some useful probabilistic distributions and their relationships. Compound distributions.
3. **Models for the counting process.** Poisson process (homogeneous and inhomogeneous). Renewal process. Delayed renewal process. Mixed Poisson process. Compound Poisson process.
4. **Claim size models.** Heavy-tailed and light-tailed distributions. Regularly varying functions and distributions. Subexponential distributions.

5. **Total claim amount.** Mean, variance and asymptotic behavior in the Sparre - Andersen model. Classical premium calculation principles: pure premium (equivalence) principle, expected value principle, the variance principle, standard deviation principle, the principle of zero utility, the Esscher principle, risk adjusted premium principle. Calculation and approximation of distribution of total claim amount.
6. **Ruin probabilities.** Risk process, ruin probability and net profit condition. Bounds for the Ruin Probability. Lundberg's inequality. Pollaczeck - Khinchine formula. Exact asymptotics for the ruin probability for small and large claim sizes.
7. **Reinsurance.** Proportional Reinsurance. Excess - of - Loss Reinsurance. Stop - Loss Reinsurance.

Method of assessment

The final grade is based on the results of home tasks solving and on oral exam. Course program contains sample tasks, exam questions and the detailed grade for this exam.

Main readings

1. Mikosh T. *Non - life insurance mathematics. An Introduction with the Poisson Process*, 2nd ed. Springer-Verlag Berlin Heidelberg, 2009.
2. Rolski, T., Schmidli, H., Schmidt, V. and Teugels, J. *Stochastic Processes for Insurance and Finance*, John Wiley & Sons, Chichester, 1999
3. Asmussen S. *Ruin Probabilities*. Singapore: World Scientific Publishing Co., 2000.
4. Bowers, N. L., Gerber, H. U., Hickman, J. C., Jones, D. A., and Nesbitt, C. J.: *Actuarial Mathematics*. 2nd ed., Society of Actuaries. Schaumburg, Illinois, 1997.

Title of the course:
M.2-E-3 Applied Regression Analysis

Information about the author:
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Course description

Regression models are widely used in scientific and technical research. However, their use raises the question: have you selected the correct regression model? Do the real data satisfy probabilistic assumptions which are supposed in the regression model? The first thing that explorer has to take care is picking the right covariates. This selection is done in different ways depending on the intended set of possible covariates. In particular, there are different algorithms for power and trigonometric regression functions. After the selection have done, the regression model should be checked for compliance with the real data. The required statistical tests are developed since the 1970's. These tests are based on the weak convergence in C (0,1) of process of partial sums of regression residuals to a centered Gaussian process. Then the limiting process is known, the researcher selects a functional of the initial process and calculates a p-value for the data.

Limiting Gaussian process depends strongly on the properties of the regression model. Here we distinguish three situations. If regression errors have a normal distribution, the researcher calculates sequential regression residuals, and the process of their sums converges to a standard Wiener process. If the errors are not normal and the covariates are values of deterministic functions at fixed points then MacNeill theorem and its generalizations are used. If the errors are not normal and the covariates are order statistics or induced order statistics then Davydov-Egorov theorem about asymptotic behavior of the process sums of induced order statistics is used. We derive algorithms of analysis of the adequacy of regression models in each of these situations. Each algorithm is illustrated with practical examples.

Learning outcomes of the course

As the result of study of the course "Applied Regression Analysis" the student
should know the basics of theory and applications of statistical testing of regression models;
should be able to analyze and investigate (theoretically and numerically) regression models in terms of their reliability and possibility of their use in applications;
should possess the technologies for construction of statistical tests of regression models.

Course content

1.	**Estimating in regression models**
1.1	The main problems in which regression models are used. A problem of estimating of sample evidence as a simplest problem of regression estimating
1.2	A least-squared method and a problem of normal regression
1.3	Gauss-Markov theorem
1.4	Significance of coefficients in polynomial models
1.5	Significance of coefficients in trigonometric polynomial models
1.6	Problems of estimating in non-linear regression models
2.	**Sums of residuals of regression models of time series**
2.1	Equivalent conditions of weak convergence in C(0,1). Functional Central Limit Theorem (Invariance Principle)
2.2	Distributions of some functionals of standard Brownian motion
2.3	Recursive residuals for normal regressions
2.4	MacNeill theorem
2.5	A generalization of MacNeill theorem and its consequences
2.6	An analogue of MacNeill theorem in a non-linear case
3.	**Sums of residuals of regression on order statistics**
3.1	Applications of regression on order statistics. Some properties of order statistics. Asymptotics of partial sums.
3.2	One-parameter linear regression against order statistics. Simple regression
3.3	Induced order statistics, its properties. Davydov-Egorov theorem
3.4	A general theorem about asymptotics of sums of residuals of a linear regression on induced order

	statistics
3.5	Propagation of the general theorem to a non-linear case
3.6	Corollaries of the general theorem for concrete models

Method of assessment

In the program of the course, the carrying out the examination is provided. The detailed grade for this examination is presented in the working program of the discipline **M.2-E-3**.

Basic literature

Kovalevskii A.P. Applied Regression Analysis. Novosibirsk, 2013 (electronic version).

Title of the course:

M.2-E-4 Financial Mathematics

Information about the author:

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Course description

Financial Mathematics is a branch of mathematics designed to analyze the financial structure, operating under conditions of uncertainty and find the most rational ways to manage financial institutions and facilities, taking into account such factors as time, risk, stochastic evolution, etc.

The main objective of financial mathematics is to provide an adequate assessment tools, taking into account the probabilistic nature of market conditions and the flow of payments from the instruments. Also methods of assessment of financial risks are based on the models of financial mathematics. In the stochastic financial mathematics it is necessary to define the criteria for risk assessment, including an adequate assessment of the financial instruments.

Financial markets are the main object of Financial Mathematics, based on such probabilistic and statistical disciplines as Stochastic Processes, Statistics of Stochastic Processes, Martingale Theory, Stochastic Analysis, etc.

The *goals* and *objectives* of the course:

• to give basic concepts and results of Stochastic Financial Mathematics;
• to give applications to a variety of estimates in a stochastic financial engineering;
• to develope the basic knowledge of stochastic processes in economics and finance;
• to acquaint with the stochastic analysis and calculations in models of financial markets, which operate under conditions of uncertainty;
• to acquaint with practical skills in the using of stochastic methods for the calculation of the corresponding continuous econometric models;
• to acquaint with the ability to interpret mathematical results for the prediction and explanation of economic effects and management of economic systems.

Learning outcomes of the course

As the result of study of the course “Financial Mathematics” the student
should know basic concepts and results of Stochastic Financial Mathematics*;*
should know applications to a variety of estimates in a stochastic financial engineering;
should know the stochastic analysis and calculations in models of financial markets, which operate under conditions of uncertainty;
should possess methods of the stochastic analysis for solving test and advanced problems of economics and finance.

Course content

The course consists of four chapters. The first chapter presents the basic concepts and tasks of Financial Mathematics. In the second chapter we study the stochastic pricing model. In particular, we will consider the classic options: European options, Asian options, «lookback» options. The third chapter is an introduction to Stochastic Calculus. In particular, in the third chapter we study Ito integral - the main tool of the theory of the Stochastic Calculus.

In the fourth chapter we examine the calculation of the cost of derivatives in the continuous case and investigate in detail the Black-Scholes model.

1. The basic concepts and tasks of financial mathematics

The subject of financial mathematics. Examples of contracts. Arbitration. Hedging. Optimal investment prices payment obligations. Conditional expectations: definition, existence and uniqueness, basic properties.

Martingales: discrete and continuous time. Martingales, submartingales, supermartingales. Definitions and basic properties. Examples. Doob decomposition. Stopping times. Upcrossings. Basic inequalities. Convergence theorems. Uniformly integrable martingales. The discrete version of the stochastic integral. (B, S)-market and investment portfolio. The condition of self-financing. Discounting. The first and second fundamental theorem of financial mathematics. Incomplete markets. Upper hedging price.

2. Stochastic models for pricing

The concept of no-arbitrage in the market. The pricing model of financial assets. Arbitrage pricing theory. The calculation of arbitrage (fair) value of classic options: European options, Asian options, «lookback» options. The problem of optimal investment: a martingale approach. American options. Supermartingale characterization of the cost. The optimal time of the execute of the American options.

3. Introduction to stochastic calculus

Brownian motion (Wiener process): definition and basic properties. The construction of the continuous Brownian motion. Properties of the trajectories. The quadratic variation of the Brownian motion. The distribution of the maximum of the Brownian motion. Geometric Brownian motion.

The construction of the Ito stochastic integral. Properties of the stochastic integral. Ito’s change of variables formula. Ito process. The quadratic variation of the Ito integral and Ito process. Brownian bridge. Representation of geometric Brownian motion as an Ito process. Stochastic differential equation. Weak and strong solutions.

4. The calculation of the value of derivatives in the continuous case.

The Black-Scholes model. The Black-Scholes equation and its solution.

The Black-Scholes formula as a limiting case of the discrete Cox-Ross-Rubinstein formula. The Black-Scholes formula as the solution of a stochastic differential equation with the boundary conditions for a European call and put options. Girsanov theorem for the Brownian motion. Risk-neutral measure and Girsanov theorem in the general case. The derivation of the Black-Scholes using the Girsanov theorem.

Asian type options, "barrier" options. «Lookback» options. American call and put options. Inequalities relating the options of European and American types.

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. *Shiryaev A. N.* Probability. New York, NY: Springer-Verlag, 1995.
2. *Shiryaev A. N.* Stochastic financial mathematics, 2002.
3. *Oksendal B.* Stochastic Differential Equations, New York, NY: Springer-Verlag, 2003.
4. *Baxter M. W., Rennie A J. O.* Financial Calculus. An introduction to derivative pricing. Cambridge University Press, Cambridge 2001.
5. *Brzeźniak Z., Zastawniak T. J.* Basic Stochastic Processes: A Course Through Exercises. Springer, 1999.
6. *Shreve S.* Stochastic Calculus for Finance I, II. Springer, 2004.
7. *Steele M.* Stochastic Calculus and Financial Applications. Springer, 2001.

Title of the course:

M.2-E-5 Markov Chains

Information about the author:

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Course description

The course “Markov Chains” is an elective course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP “Numerical Statistical Modelling and Simulation. Monte Carlo Methods”, “Mathematical and Computer Modelling in Mechanics” , “Modern Trends in Discrete Mathematics and Combinatorial Optimization” of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability Theory and Stochastic Processes and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - “Mathematics”. As a prerequisite, a modest understanding of Probability theory at an undergraduate level and of some basic notions in Stochastic Processes is assumed. The course substantially deals with discrete-time Markov models both in discrete state space and in general state space.

The course contains main notations and definitions, introduce class structure for Markov Chains with discrete state space, discusses hitting times and absorption probabilities, recurrence and transience and limit theorems on the conditions of convergence to an equilibrium distribution for the chains with discrete state space. For Markov Chains in general state spaces, the various threads are possible. The strand chosen by this course mainly follows selected topics of famous S.P. Meyn and R.L. Tweedie book “Markov Chains and Stochastic Stability” and looks at the Markov chains in general state spaces through the prism of regeneration.

The aim of the course “Markov Chains” is to acquaint students with actual theoretical aspects of Markov chains theory, to provide student with the knowledge and ability to apply modern methods of research in the analysis of Markov models arising in applications.

The *objectives* of the course “Markov Chains” are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics.

Learning outcomes of the course

As the result of studying the course “Markov Chain” the student
should understand the role and place of this discipline among the other mathematical subjects; *should know* the definitions, basic properties and examples of Markov chains and their practical value;
should be able to apply their knowledge to solve mathematical problems.

Course content

1. Countable Markov chains. Definition and basic properties. Examples. Time-homogeneity. Random times and strong Markov property. Class structure. Hitting times. Recurrence and transience. Invariant distributions. Limiting behavior of recurrent Markov chains. Transformations of Markov chains.

2. Harris Markov chains. Basic definitions. Occupation, hitting and stopping times. Irreducibility. Irreducible models on a countable space. Irreducibility on general state space. Minorization condition. Small sets. Harris recurrence. Existence of invariant measure. Ergodicity. Geometric ergodicity.

Method of assessment

The final grade is based on the results of oral exam. Course program contains sample exam questions and the detailed grade for this exam.

Main reading

1. J. G. Kemeny and J. L. Snell. *Finite Markov Chains.* Van Nostrand, Princeton, N.J. 1960.
2. O.Hernández-Lerma, J.B. Lasserre. *Markov Chains and Invariant Probabilities.* Birkhauser Verlag AG, 2003.
3. S. P. Meyn and R.L. Tweedie. *Markov Chains and Stochastic Stability.* Cambridge University Press, 2009.

Title of the course:

M.2-E-6 Theory of V-statistics

Information about the author:

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Course description

The elective course "Theory of V-statistics" is one of the possible continuations of the basic courses "Probability Theory" and "Mathematical Statistics" (training direction 010100 "Mathematics") and substantially deals with limit theorems and asymptotic behavior of so-called normalized V-statistics (and U-statistics as well). These objects generalize sums of random variables and are very popular in applied statistics.

The course contains main notions and definitions, classifications of U- and V-statistics, limit theorems for them (in different situations) and their proofs as well as proofs of some important auxiliary statements from certain areas of the probability theory and analysis (multidimensional L_2-spaces, stochastic integrals, weak dependence of random variables).

The course also contains up-to-date results concerning exponential bounds of distribution functions of V-statistics as well as limit elements which arise in theorems describing the asymptotic behavior of sequences of normalized "canonical" V- (or U-) statistics.

The *aim* of the course "Theory of V-statistics" is to acquaint students with actual theoretical and applied aspects of stochastic analysis and asymptotic methods in mathematical statistics.

The corresponding *objectives* of the course "Theory of V-statistics" are:

- to give to students advanced knowledge in the theory and applications of modern mathematical statistics and probability theory;
- to teach and train students to formulate and solve theoretical problems in certain areas of modern mathematical statistics and probability theory;

When developing the course, the author's experience in teaching at Mechanical Mathematical Department of NRU NSU the disciplines related to the problems of mathematical statistics and probability theory together with author's teaching aids and recent scientific articles, were used.

The course "Theory of V-statistics" can be used in realization of the international master programme "Probability and Statistics" of DMM NRU NSU.

Learning outcomes of the course

As the result of studying the course "Theory of V-statistics" the student

should know basic properties, examples and classifications of U- and V-statistics;

should be able to formulate and prove theorems describing asymptotic behavior of U- and V-statistics, their properties and properties of the limit elements;

should be able to use the asymptotical methods for solving test and actual problems of the modern mathematical statistics.

Course content

1. U-statistics and V-statistics. Hoeffding decompostion.
2. Asymptotic normality of non-degenerate U- and V-statistics
3. Multiple Wiener-Ito integrals. Diagram formula. Limit theorems for canonical (degenerate) V-statistics in case of independent observations.
4. Construction of stochastic integral without orthogonality of the integrating measure.
5. Limit theorems for canonical V-statistics in case of stationary sequence of observations with weak dependency conditions. Limit elements as multiple stochastic integrals.
6. Limit elements as multivariate orthogonal series of Gaussian random variables.
7. Exponential inequalities.

Method of assessment

In the program of the course, the carrying out the examination is provided. The detailed grade for this examination is presented in the working program of the discipline M.2-E-6.

Basic literature

1. Bystrov A. A. "Theory of V-statistics". Novosibirsk, 2013 (electronic version).
2. Borovskich Yu. V. and Koroljuk V. S. "Theory of U-statistics". Kluwer, Dordrecht, The Netherlands, 1994.
3. Major Peter. On the estimation of multiple random integrals and U-statistics (Lecture Note). August 2013, Springer.

Title of the course:

M.2-E-7 Statistics of Stochastic Processes

Information about the author:

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— Doctor of physical and mathematical sciences
— Professor of the Chair of Probability Theory and Mathematical Statistics at the Department of Mechanics and Mathematics of Novosibirsk State University (NSU); see http://www.nsu.ru/mmf/tvims
— Leading Researcher of the Laboratory of Probability Theory and Mathematical Statistics at the Sobolev Institute of Mathematics of Siberian Branch of RAS; see http://math.nsc.ru/LBRT/v1/dima/dima.html
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Course description

Original lecture course Statistics of Stochastic Processes was developed for teaching foreign students studying Probability and Statistics in English. This is an elective course in the main (variable) section of professional cycle of disciplines of Master Educational Programme (MEP) "Probability and Statistics". This course can also be used in MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods", "Mathematical and Computer Modelling in Mechanics" , "Modern Trends in Discrete Mathematics and Combinatorial Optimization" of Department of Mechanics and Mathematics

NRU NSU. This course is based on the core courses on Probability Theory and Stochastic Processes and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - "Mathematics". As a prerequisite, a modest understanding of Probability theory at an undergraduate level and of some basic notions in Stochastic Processes is assumed.

The main part of the course is concerned with the theory and applications of stationary processes Their spectral representation is deduced by methods of Hilbert space geometry introduced at the beginning of the course. The course is concluded with discussion of the problems of prediction and filtering.

The aim of the course "Statistics of Stochastic Processes" is to acquaint students with actual theoretical aspects of statistics of stochastic processes theory, to provide student with the knowledge and ability to apply modern methods of research in the analysis of statistics of stochastic processes arising in applications.

The *objectives* of the course "Statistics of Stochastic Processes" are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics.

Learning outcomes of the course

As the result of study of the course "Statistics of Stochastic Processes" the student
should understand the role and place of this discipline among the other mathematical subjects;
should know the definitions, basic properties and examples related to the Statistics of Stochastic Processes and their practical value;
should be able to apply their knowledge to solve mathematical problems.

Course content

1. The Hilbert space $L_2(\Omega, F, \mathbf{P})$. Mean value of the process and correlation (covariace) function. Covariance function of the Wiener and Poisson processes.
2. Continuity of the stochastic process in L_2. Criterion for continuity. Continuity of the Wiener and Poisson processes in L_2.
3. Differentiability of the stochastic process in L_2. Non-differentiability of the Wiener and Poisson processes in L_2.
4. Riemann integral for stochastic process in L_2.
5. Stationary stochastic processes and sequences. Positive definiteness of the covariance function. Stationarity of the Wiener and Poisson processes.
6. Gaussian distributions.
7. Gaussian process. The existence of a Gaussian process with covariance function coinciding with given positively definite funstion B(t,s). The existence of a Gaussian process with covariance function coinciding withgiven positively definite function b(t).
8. Elementary stochastic orthogonal measure. Structure function of stochastic measure. Stochastic integral of non-random function.
9. Distribution of the integral of non-random function with respect to the wiener measure.
10. Physic content of the spectral theory.
11. Spectral measure of a stationary sequence. The notion of the spectral density.
12. Spectral representation of a stationary sequence.
13. Shannon-Kkotelnikov formula.
14. Examples of stationary sequences and their specters.
15. Problem of prediction of random sequence.

16. Deterministic stochastic sequence. Criterion for determinacy of a stochastic sequence.
17. Purely nondeterministic stochastic sequence. Criterion for nondeterminacy of a stochastic sequence.
18. Therem on decomposition of a stationary sequence into deterministic and purely nondeterministic components.
19. Precise prediction for deterministic sequences.
20. Theorem on representation of a purely nondeterministic stochastic sequence as a moving overage.
21. Theorem on general representation of the spectral density of purely nondeterministic stochastic sequence.
22. Prediction of purely nondeterministic stochastic sequences (Wiener's method).

Method of assessment

Oral exam.

Literature

1. Borovkov, A. A.: Probability Theory. Nauka, Moscow (1986)
2. Cramer, H., Leadbetter, M. R: Stationary and Related Stochastic Processes: Sample Function Properties and Their Applications. Dover (2004)
3. Yurinskii, V. V.: Stochastic Processes. Novosibirsk, NSU (1987)

Title of the course:

M.2-E-8 Limit Theorems for Sums of Multivariate Random Variables

Information about the author:

Borisov Igor Semenovich

— Professor, Doctor of physical and mathematical sciences;
— Professor of Chair of Probability and Statistics in DMM NSU;
— Leading researcher of IM SB RAS.

Course description

The course provides a basis technique to describe limit behavior of the distributions of sums of independent or weakly dependent random variables taking values in multidimensional linear normed spaces. There are studied the following two types of limiting approximation of the sum distributions: Gaussian and Poissonian ones. A number of applications and examples are considered, especially, in Mathematical Statistics. Some recent results of the author are also introduced in this course.

Learning outcomes of the course

As the result of study of the course “Limit Theorems for Sums of Multivariate Random Variables” the student
- should know the basics of theory and applications of the limit theory for the distributions of sums of random variables in infinite dimensional linear spaces;

- should know some modern concepts of this limit theory;
- should be able to use methods of the theory to solve scientific and applied problems.

Course content

1 Introduction: Donsker's Theorem, Metric Entropy, and Inequalities
1.1 Empirical processes: the classical case
1.2 Metric entropy
1.3 Inequalities

2 Gaussian Measures and Processes
2.1 Sample Continuity
2.2 Gaussian vectors
2.3 Inequalities and comparisons for Gaussian distributions

3 Foundations of Uniform Central Limit Theorems: Donsker Classes
3.1 Convergence in law
3.2 Almost uniform convergence amd convergence in outer probability
3.3 Almost surely convergent realizations
3.4 Unions of Donsker classes
3.5 Sequences of sets and functions

4 Vapnik-Cervonenkis Combinatorics
4.1 Vapnik-Cervonenkis classes
4.2 Maximal classes
4.3 Classes of index 1
4.4 Combining VC classes
4.5 Vapnik-Cervonenkis properties of classes of functions

5 Limit Theorems for Vapnik-Cervonenkis and Related Classes
5.1 Koltchinskii-Pollard entropy and Glivenko-Cantelli theorems
5.2 Vapnik-Cervonenkis-Steele laws of large numbers
5.3 Pollard's central limit theorem
5.4 Necessary conditions for limit theorems
5.5 Inequalities for empirical processes
5.6 Glivenko-Cantelli properties and random entropy

6 Metric Entropy, with Inclusion and Bracketing
6.1 Definitions
6.2 Central limit theorems with bracketing
6.3 The power set of a countable set: the Borisov-Durst theorem

7 Approximation of Functions and Sets
7.1 Introduction: the Hausdorff metric
7.2 Spaces of differentiable functions and sets with differentiable boundaries
7.3 Lower layers
7.4 Metric entropy of classes of convex sets

8 Sums in General Banach Spaces and Invariance Principles

8.1 Independent random elements and partial sums
8.2 A finite-dimensional invariance principle
8.3 Invariance principles for empirical processes
8.4 Log log laws and speeds of convergence

9 Universal and Uniform Central Limit Theorems
9.1 Universal Donsker classes
9.2 Metric entropy of convex hulls in Hilbert space
9.3 Uniform Donsker classes

10. Poissonian approximation of sums of independent random variables in linear spaces
10.1 The classical Poisson limit theorem
10.2 Couplings in the Poissonian approximation
10.3 Speed of convergence
10.4 Poissonian stochastic processes
10.5 Compound Poisson distributions
10.6 Poissonization

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. R.M.Dudley, *Uniform Central Limit Theorems*, Cambridge University Press, 2004.
2. I.S.Borisov, *Gaussian and Poissonian approximations for sums of random vectors*, Novosibirsk STATE university, Novosibirsk, 2013 (electronic version).

Title of the course:

M.2-E-9 Asymptotic Analysis of Functionals of Order Statistics

Information about the author:

Baklanov Evgeny Anatol’evich
- Candidate (PhD) of physical and mathematical sciences;
- Associate Professor of the CPTMS DMM NSU;
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Course description

The elective course “Asymptotic Analysis of Functionals of Order Statistics” is one of the possible continuations of the basic courses Probability Theory and Mathematical Statistics and substantially deals with limit theorems and asymptotic behavior of functionals of order statistics. These objects generalize sums of random variables and are very popular in applied statistics.

The course contains main notions and definitions, classifications of functionals of order statistics, limit theorems and their proofs as well as proofs of some important auxiliary statements from certain areas of the probability theory and analysis.

The *aim* of the course "Asymptotic Analysis of Functionals of Order Statistics" is to acquaint students with actual theoretical and applied aspects of theory of order statistics and asymptotic methods in mathematical statistics.

The corresponding *objectives* of the course Asymptotic Analysis of Functionals of Order Statistics" are:

- to introduce students to the basic concepts and methods of modern theory of order statistics;
- to give to students advanced knowledge in the theory and applications of modern mathematical statistics and probability theory;
- to teach and train students to formulate and solve theoretical and practical problems in certain areas of modern mathematical statistics and probability theory.

When developing the course, the author's experience in teaching at Mechanical and Mathematical Department of NRU NSU the disciplines related to the problems of mathematical statistics and probability theory together with author's teaching aids and recent scientific articles, were used. The course "Asymptotic Analysis of Functionals of Order Statistics" can be used in realization of the international master program "Probability and Statistics" of DMM NRU NSU.

Learning outcomes of the course

As the result of studying the course "Asymptotic Analysis of Functionals of Order Statistics" the student

should know basic properties, examples and classifications of functionals of order statistics;

should be able to formulate and prove theorems describing asymptotic behavior of linear functions of order statistics;

should be able to use the asymptotical methods for solving test and actual problems of the modern mathematical statistics.

Course content

1. Introduction to Theory of Order Statistics

Basic distribution theory. Some properties of order statistics. Distribution of the median, range, and some other statistics. Exercises. Discrete order statistics. Moment relations, bounds, and approximations. Asymptotic theory.

2. L-statistics

Moment inequalities. Exponential inequalities. Limit theorems for classical L-statistics. Asymptotic normality of linear functions of order statistics.

3. Functionals of Order Statistics

Additive functionals of order statistics, examples. Lorenz-type curves. Weak dependence conditions. Strong laws of large numbers for L-statistics based on dependent data. Limit theorems for the Lorenz-type curves.

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. *David, H. A., Nagaraja H. N.* Order Statistics. 3rd ed. Wiley, 2003.
2. *Baklanov, E. A.* The strong law of large numbers for L-statistics with dependent data. Sib. Mat. Zh. 47, No. 6, 2006.
3. *Baklanov, E. A.,Borisov, I. S.* Probability inequalities and limit theorems for generalized L-statistics. Lith. Math. J. 43, No. 2, 2003.

Title of the course:
M.2-E-10 Regression Analysis

Information about the author

Linke Yuliana Yurievna
— Candidate (PhD) of physical and mathematical sciences;
— Associate professor of Chair of Probability and Statistics in DMM NSU;
— researcher of IM SB RAS.

Course description

Regression analysis consists of techniques for modeling the relationship between a dependent variable and one or more independent variables. Regression analysis is one of the most commonly used statistical methods in practice. Applications of regression analysis can be found in many scientific fields including medicine, biology, agriculture, economics, engineering, sociology, geology, etc. The course "Regression Analysis" involves the basic models and concepts of the linear and nonlinear regression theory. The recent theoretical results of the author are also introduced in this course.

Learning outcomes of the course

As the result of study of the course "Regression Analysis" the student
— should know the basics of theory and applications of regression analysis;
— should know some modern concepts of regression methods;
— should be able to use regression methods for solving of scientific and applied problems.

Course content

1. INTRODUCTION. Regression Models. Goals and problems of regression analysis.

2. REVIEW OF SIMPLE REGRESSION. The linear model and assumptions. Least squares estimation. Statistical properties of the least squares estimation. Geometry of least squares (sums of squares and degrees of freedom; reparameterization; the collinearity problem). Maximum likelihood estimation. Predicted values and residuals. Analysis of variation in the dependent variable. Precision of estimates. Tests of significance and confidence intervals. Regression through the origin.

3. MULTIPLE REGRESSION IN MATRIX NOTATION. The model. The normal equations and their solution. The residuals vectors. Properties of regression estimates. The least squares estimates of multiple regression parameters under linear restrictions.

4. HYPOTHESIS TESTING AND CONFIDENCE REGIONS. The general linear hypothesis. Test for general linear hypothesis. Special cases of the general form. Sequential and partial sums of squares. Univariate and joint confidence regions. Simultaneous confidence statements. Joint confidence regions. Confidence intervals of mean and prediction in multiple regression.

5. MODELS NONLINEAR IN THE PARAMETERS. Examples of nonlinear models. Fitting models nonlinear in the parameters. Inference in nonlinear models. Violation of assumptions. Heteroscedastic errors. Correlated errors. Logistic regression. Polynomial regression.

6. PROBLEM AREAS IN LEAST SQUARES. Nonnormality. Heterogeneous. Variances. Correlated errors. Influential data points and outliers. Model inadequacies. The collinearity problem. Errors in the independent variables. Heteroscedasticity.

7. MEASUREMENT ERROR MODELS. Measurement error examples. Effects of measurement error. Ordinary least squares and measurement error. The functional model. The structural model. Berkson errors. Bias caused by measurement error. Method of correcting for bias. Methodology and asymptotic properties. Approximations using replicate data. Other methodologies: Computation and asymptotic approximations.

8. HETEROSCEDASTICITY. Two-step estimation methods in linear and fraction-linear regression. Two-step estimation methods in error-in-variable models.

9. OTHER ASPECTS. Shrinkage methods. Variable selection. Analysis of variance and covariance. Test of homogeneity of regression coefficients. Residuals analysis. Transformation of variables (transformations to simplify relationships, transformations to stabilize variances, transformations to improve normality). Collinearity problems (analysis of the correlational structure, principal component regression). Analysis of unbalanced data. Mixed effects models.

Method of assessment

In the program of the course, the carrying out the examination is provided.

Basic literature

1. Linke Yu.Yu. Statistical Regression.Modern Aspects. Novosibirsk, 2013 (electronic version).

Title of the course:

M.2-E-11 Boundary Crossing Problems for Random Walks

Information about the author:

Lotov Vladimir Ivanovich

- Full Professor, Doctor of physical and mathematical sciences;
- Head of the Chair of Probability Theory and Mathematical Statistics at the University;
- Chief scientist at the laboratory of the Probability Theory and Mathematical Statistics of the Sobolev Institute of Mathematics;
- see also the section “Information about the supervisor of the Programme” of this Programme;
- e-mail: lotov@math.nsc.ru

Course description

The course provides an introduction to an important part of modern Probability Theory related to the study of various distributions connected with mutual disposition of a random walk trajectory and boundary of certain set. Such problems arise in Statistics, Risk Theory, Queueing Theory, etc. We study in detail properties of the first passage time and overshoot in the problem with one boundary and give an introduction to a so called factorization method. A special attention is paid to finding factorization components in an explicit form. We analyze the distribution of supremum of a random walk trajectory and study its asymptotic behavior. Random walks with two boundaries are studied by using factorization method. It includes the identity of Kemperman, Wiener-Hopf method for solving this identity and finding explicit expressions and asymptotic representations for the distribution of overshoot, number of crossings, etc.

Learning outcomes of the course

As the result of studying the course “Boundary crossing problems for random walks” the student

should know basic properties of trajectories of a random walk, main factorization identities, properties of the distribution of supremum, factorization method of finding moment generating functions in boundary problems;

should be able to formulate and prove theorems describing basic properties of random walks;

should be able to use factorization method and asymptotic analysis of basic distributions in boundary crossing problems.

Course content

1. The concept of the "Boundary crossing problems".
2. Applications of boundary crossing problems.
3. Ladder epochs and ladder heights.
4. First passage time and overshoot. A property of the exponential distribution.
5. Finiteness of moments for the first passage time.
6. Factorization identities.
7. Analysis of general properties of random walks based on factorization identities.
8. Explicit expressions for factorization components.
9. Characteristic function for the supremum of a random walk.
10. The asymptotics of the distribution of the supremum.
11. Random walks with two boundaries. The ruin problem.
12. An identity of Kemperman.
13. Factorization operators in boundary crossing problems and their properties.
14. The study of the ruin probability, number of crossings, and sojourn time by means of factorization operators.
15. Asymptotic analysis in boundary crossing problems.

Method of assessment

In the program of the course, the carrying out the examination is provided. The detailed grade for this examination is presented in the working program of the discipline M.2-E-11.

Basic literature

Borovkov A.A. Probability Theory. Heidelberg: Springer-Verlag, 2013.
Borovkov A.A. Stochastic Processes in Queueing Theory. Springer, 1976.
Gut A. Stopped random walks. Springer, 2009.
Lotov V.I. Limit theorems in a boundary crossing problems for random walks. Siberian Math. J., V.40, N.5, 925-937.

Title of the course:

M.2-E-12 Queueing Theory

Information about the author:

Chernova Natalia Isaakovna

- Candidate (PhD) of physical and mathematical sciences;
- Associate professor of the Chair of Probability and Statistics of Department of Mechanics and Mathematics (DMM) of Novosibirsk State University (NSU);
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Course description

The course “Queueing Theory” is an elective course in the main (variable) section of professional cycle of Master Educational Programme (MEP) “Probability and Statistics”. This course can also be used in MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods", "Mathematical and Computer Modelling in Mechanics" , "Modern Trends in Discrete Mathematics and Combinatorial Optimization" of Department of Mechanics and Mathematics NRU NSU. This course is based on the core courses on Probability Theory and Stochastic Processes and satisfy the requirements of State Master Standard (SMS) for training direction 010100 - "Mathematics". The course focuses on modern models and methods of Queueing theory. Queueing theory has numerous applications in telecommunications, traffic control, predicting computer performance, health services, airport traffic, the mining industry, layout of manufacturing systems etc. Recently, the interest in Queueing theory has been revived, not only as a result of new, related applied problems, but also due to the emergence of new mathematical approaches to their solution.

The course can serve as an introduction to modern queueing theory and treats in detail basic structures like Markovian models, GI/G/1 and GI/G/s queues and queueing networks. An overview of advanced research tools and the very latest developments is also provided. The course highlights a number of useful approaches to establishing stability of queueing systems: Borovkov's renovating method, saturation rule and fluid approximation approach. Some special queueing models (polling models etc) that have attracted considerable attention in practice, serve as examples.

The *aim* of the course "Queueing Theory" is to introduce students to relevant theoretical aspects of this branch of science, to provide the student with the knowledge and ability to apply modern methods of investigation the behavior of queueing systems and networks arising in applications.

The *objectives* of the course "Queueing Theory" are to provide students with advanced knowledge of basic concepts in the subject matter, to supply students by sound knowledge of modern methods of research in the area, to train student's ability to identify, summarize, formulate and solve theoretical problems in certain areas of mathematics.

Learning outcomes of the course

As the result of studying the course "Queueing Theory" the student
should understand the role and place of this discipline among the other mathematical subjects;
should know the practical value of Queueing theory;
should be aware of the fundamental resources and notation within the Queueing theory;
should understand the fundamental queueing relations;
should be able to apply their knowledge to solve theoretical and practical problems.

Course content

1. Introduction and Examples.
2. Basic probability tools. Probability generating function, Laplace-Stieltjes transform, some useful probability distributions (Poisson, Geometric, Exponential, Erlang, Hyperexponential). Poisson process.
3. Basic notions of Queueing theory. Kendall notation. Little's Law.
4. Birth-and-Death queueing model. Markovian models: M/M/1-Queue, M/M/n-Queue, Finite M/M/1/K-Queue.
5. General methods. Renewal Theory. Regenerative processes. Random walks and Wiener-Hopf factorization. The properties of supremum of random walk and related problems of Queueing theory. Renovating method. Saturation rule. Fluid approximation approach.
6. Queues with Vacations, Priority Queues, Open Jackson networks, Loss systems, Polling systems.

Method of assessment

The final grade is based on the results of oral exam. Course program contains sample exam questions and the detailed grade for this exam.

Main readings

1. U. Narayan Bhat. An introduction to queueing theory: Modeling and analysis in applications. Birkhäuser Boston, 2008.
2. A.A.Borovkov. Asymptotic methods in queueing theory. J. Wiley, 1984, 292 p.
3. S.Asmussen. Applied probability and queues. Springer, 2003, 438 p.
4. Ivo Adan and Jacques Resing. Queueing Theory. Department of Mathematics and Computing Science, Eindhoven University of Technology, 2001.
5. N.I.Chernova. Ergodicity of Queueing Models. Novosibirsk 2013 (electronic version).

Title of the course:

M.2-E-13 Advanced stochastic numerical methods in applied mathematics and physics

(course of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods")

Information about the author:

Karl Karlovich Sabelfeld

- Full Professor, Doctor of physical and mathematical sciences;
- major researcher of LSP ICM&MG SB RAS; see http://osmf.sscc.ru/mixa/bak.html;
- head of the Laboratory "Center of Energy Efficiency Technologies" NRU NSU;
- editor-in-chief of the international journal "Monte Carlo Methods and Applications, de Gruyter, Berlin;
- see also the section «Information about the author» of working program of the discipline **M.2-V-8**;
- e-mail: sabelfeld.karl@yahoo.de, karl@osmf.sscc.ru

Course description

The course "Advanced stochastic numerical methods in applied mathematics and physics" presents the modern stochastic models and randomization based numerical methods and algorithms for solving large scale problems of computational mathematics with applications in physics, chemistry, biology and different fields of scientific computing. Mainly two basic approaches are given: (1) direct stochastic simulation of the processes in question which approximately mimics the physics and reflects properly the main features, e.g., diffusional and ballistic transport of particles in random porous media, annihilation of electrons and holes in semiconductors, growth of nanowires, coalescence of growing islands of adatoms, simulation of random turbulent velocity fields, x-ray diffraction analysis of dislocations in crystals, luminescence in nanowires, and many others, and (2) development of stochastic models and simulation methods for solving boundary value problems, both in deterministic and probabilistic formulation, e.g., random walk based methods for solving the diffusion and electrostatic problems, the elastostatics problems with random boundary conditions, Darcy equation with random hydraulic conductivity, nonlinear Smoluchowski equation governing the coagulation, diffusion, trapping and annihilation, and many others, as well as solution of direct and inverse problems based on randomization of large matrices and a randomized SVD (singular value decomposition) and stochastic projection methods.

Learning outcomes of the course

The student is assumed to get the advanced stochastic simulation instruments, and should be able to apply the described techniques to scientific and engineering problems including both direct simulation and random walk methods for solving boundary value problems.

Course content

1. Introducting information to the course presented and basic principles of randomization
2. Random walks and diffusion. Laplace and the heat equations
3. Randomization of matrices. Markov chains and products of matrices
4. Integral operators and discrete approximations. Integral equations, von Neumann series and standard Monte Carlo estimators
5. Randomized Nystroem method for integral equations
6. Random projection methods and the Johnson-Lindenstrauss theorem
7. Randomized simple and block Kaczmarz and Cimmino methods
8. Ill-posed problems solved by randomized projection method
9. Randomized SVD and low rank approximation for linear systems
10. Solution of the Laplace equation by randomized projection method
11. Random Walk on Boundary Methods: Laplace and Heat equations
12. Random Walk on Spheres methods: Laplace and biharmonic equations
13. Random Walk on Spheres methods: the elastostatics equations
14. Random fields simulation: the Fourier expansions for stationary processes
15. Karhunen-Loeve expansions (K-L) for non-homogeneous random fields
16. SVD based low rank approximations for random field simulation
17. Boundary value problems with random boundary conditions. Correlation analysis
18. Spatially homogeneous Nonlinear Smoluchowski equation: the mean field approach. Method of random interacting particles with markovian memory. The generator of the Markov process, and convergence of the method
19. Inhomogeneous Smoluchowski equation governing the diffusional and interacting kinetics of reacting species. Random walk on spheres method
20. Annihilation of electrons and holes with trapping centers: a two species Smoluchowski equation
21. Stochastic polynomial chaos expansion for solving boundary value problems with random coefficients
22. Solution of the Darcy equation with random lognormal hydraulic conductivity
23. Simulation of diffusion processes for domains with highly heterogeneous boundaries
24. A stochastic model for the nanowire growth simulation
25. Turbulence simulation and particles transport in the turbulent atmosphere
26. Stochastic Lagrangian models for one and two interacting particles. The footprint problem
27. Coagulation of particles in intermittent turbulent flows
28. Stochastic Lagrangian models for inhomogeneous Smoluchowski equation
29. Stochastic method of fundamental solutions for PDEs. Simulation of charged particles transport
30. Simulation of filtration processes in porous media

Method of assessment

In the program of the course, the carrying out an examination is provided. The detailed grade for this examination is presented in the working program of the discipline **M.2-V-8**.

Basic literature

1. Sabelfeld K.K. Advanced Stochastic Simulation Methods in Applied Mathematics and Physics. Novosibirsk, 2013 (electronic version).
2. Sabelfeld K.K. Monte Carlo Methods for Boundary Value Problems, Springer, Heidelberg - New York - Berlin, 1991.
3. Sabelfeld K.K. Random fields and Stochastic Lagrangian Methods. De Gruyter, Berlin, 2012.

Title of the course:

M.2-E-14 Random process simulation and continuous stochastic models

(course of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods")

Information about the author:

Sergei Mikhailovich Prigarin

- Full Professor, Doctor of physical and mathematical sciences;
- Professor of CCM DMM NSU, see http://mmfd.nsu.ru/mmf/kaf/cm/prep.asp;
- Leading Researcher of the Laboratory of Stochastic Problems (LSP) ICM&MG SB RAS, see http://osmf.sscc.ru/mixa/bak.html;
- see also the section "Information about the author" in working program of the discipline **M.1-B-2;**
- e-mail: sergeim.prigarin@gmail.com, smp@osmf.sscc.ru.

Course description

The developed course M.1-B-2 "Random process simulation and continuous stochastic models" is a fundamental discipline for section M.1-B "General cycle – basic part" of the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods" and intends for studying at the third semester of the MEP.

Computer-aided modelling is one of the most effective means which allow one to get to the root of a natural phenomenon and to predict consequences of the man's impact on the environment. Along with deterministic models stochastic models are finding ever increasing use. Recently, general methods of numerical modelling of random processes are being effectively developed and the area of application is rapidly expanded. As a matter of fact, the results obtained in the area of numerical modelling of random functions can be distinguished as a significant part of Monte Carlo methods and stochastic simulation.

The course deals with the development and investigation of numerical methods for simulation of random processes and fields. Basic concepts and methods are considered concerning construction of numerical algorithms for various types of stochastic objects. A significant part of the course is concentrated on the so-called "spectral" models. The spectral models belong to a class of continuous stochastic models. They were developed in the 70-s years of the 20th century, and have appeared to be considerably promising for various applications. In Russia, the papers by Prof. G.A. Mikhailov (the Head of the Chair of Computational Mathematics in the Novosibirsk State University) have given impetus to a large number of investigations on spectral models of random fields. Nowadays the spectral models are extensively used for stochastic simulation in the atmosphere and ocean optics, turbulence theory, analysis of pollution transport for porous media, astrophysics, and other fields of science. Moreover, great attention in the course is given to general and some specific aspects of numerical simulation of non-Gaussian models, including the problem of compatibility of marginal distributions and covariances and the method of nonlinear transformation of Gaussian functions. Up-to-date application areas of the random process modelling such as simulation of the sea surface undulation and stochastic structure of clouds are presented in the course briefly.

Various preparation levels of Russian and foreign students in probability theory, mathematical statistics, Monte Carlo methods, and stochastic processes are taken into account within the course. Extensive teaching experience of Prof. S.M. Prigarin, material of his text-books published in the Novosibirsk State University and research papers are used in the development of the course.

The course "Random process simulation and continuous stochastic models" can be used in realization of the Joint Master Programme "Applied Mathematics and Energy Strategies" (MINES Paris Tech, France – NRU NSU, Russia) and international master programs "Probability and Statistics", "Mathematical and Computer Modelling in Mechanics" and "Modern Trends in Discrete

Mathematics and Combinatorial Optimization" of DMM NRU NSU. The course "Random process simulation and continuous stochastic models" corresponds to the obligatory discipline "Modern problems of applied mathematics and information science" of the State Master Standard (SMS) for training direction 010400 – "Applied Mathematics and Information Science". Also some materials of the course concern to the obligatory disciplines "Discrete and probabilistic mathematical models" and "Continuous mathematical models" of the SMS for training direction 010400 (see the MEP "Numerical Statistical Modelling and Simulation. Monte Carlo Methods").

Learning outcomes of the course

As a result of study of the course "Random process simulation and continuous stochastic models" a student *should know* the basic concepts and methods of the random process simulation, *should be able* to construct stochastic models with required properties, and *should possess* techniques of theoretical study and numerical realization of stochastic process models.

Course content

- Introduction to Monte Carlo methods and random processes
- Types of random functions and basic approaches to stochastic simulation
- Stationary processes and spectral representations, spectral models of stationary Gaussian processes
- Spectral models for homogeneous fields on the plane and in space
- Isotropic spectral modes, spectral models of isotropic fields on a sphere
- Technique of successive refinement of spectral models on the same probability space, conditional spectral models
- Convergence of numerical models and study of errors
- Vector-valued spectral models, spectral models of potential and solenoid stochastic fields
- Non-homogeneous spectral models
- Non-Gaussian stochastic models, consistency conditions of marginal distributions and covariance functions, method of inverse distribution function
- Some applications: simulation of binary textures, imitation of atmospheric cloudiness, sea surface undulation, turbulence

Method of assessment

According to the program of the course, an examination is provided. The details on the examination are described in the working program of the discipline **M.1-B-2**.

Basic literature

1. V.A. Ogorodnikov and S.M. Prigarin, Numerical Modelling of Random Processes and Fields: Algorithms and Applications. VSP, Utrecht, the Netherlands, 1996, 240 p. [in English]
2. S.M. Prigarin, Introduction to Numerical Modelling of Random Processes and Fields. Parts 1, 2. Novosibirsk Univ. Press, 1999, 301 p. [in Russian]
3. S.M. Prigarin, Numerical Modelling of Random Processes and Fields, ICM&MG Publisher, Novosibirsk, 2005, 259 p. [in Russian]
4. S.M. Prigarin, Spectral Models of Random Processes in Monte Carlo Methods. Novosibirsk, 2013 (electronic version)

M.3 SCIENTIFIC PRACTICE AND RESEARCH WORK

Title of the discipline:

M.3-1 Scientific Seminar “Probability Theory and Mathematical Statistics”

Information about the supervisor:

Borovkov Alexander Alekseevich

- Full Member of Russian Academy of Sciences (RAS), Full Professor, Doctor of physical and mathematical sciences;
- Counselor of Russian Academy of Sciences at Sobolev Institute of Mathematics;

Discipline description

The Joint Research Seminar of the laboratory of Probability and Mathematical Statistics of Sobolev Institute of Mathematics and of the Chair of the same name of NRU NSU is the weekly workshop that is constantly working for more than 40 years. Here, the researchers of Russian and international scientific institutes and laboratories present their scientific achievements.

Learning outcomes

As the result of participating in the seminar “Probability Theory and Mathematical Statistics”, student gets the new ideas about modern concepts and applications of the Probability Theory and Mathematical Statistics. He also gets examples of presentation of reports and articles on actual scientific topics, experience in scientific discussions, asking questions, etc.

Method of assessment

The grade of the discipline “Scientific seminar “Probability Theory and Mathematical Statistics” depends on student’s attendance and activity on the seminar. The excellent mark is guaranteed to those students who are invited to give a rigorous scientific report on the seminar.

Title of the discipline:

M.3-2 Reviewing Seminar “Probability Theory and Mathematical Statistics”

Information about the supervisor:

Borovkov Alexander Alekseevich

- Full Member of Russian Academy of Sciences (RAS), Full Professor, Doctor of physical and mathematical sciences;
- Counselor of Russian Academy of Sciences at Sobolev Institute of Mathematics;

Discipline description

Reviewing Seminar "Theory of Probability and Mathematical Statistics" is designed to introduce students to the research conducted at other research centers, to instill students with the skills to run modern English-language scientific literature and public speaking skills.

To achieve this goal the following objectives of the workshop are allocated:

1) providing a quality selection of papers for abstracting;
2) assist in the organization of reports of students;
3) to discuss the reports.

Learning outcomes of the discipline

Resulting the discipline students must
be able to navigate to the main areas of research in the Probability Theory and Statistics;
be able to prepare a scientific report containing the results of its own research.

Method of assessment

The student must make a review report once during each semester. The final grade is based on the quality of the report.

Title of the discipline:

M.3-3 Preparation and defending the term paper

Description

The term paper represents scientific work which includes introduction sections of student's research work (main concepts, problem definition, plan of researches, etc.) description of the first student's scientific achievements (the proved statements, results of numerical experiments, etc.). The performance of the term paper is defined and is constantly inspected by the student's scientific supervisor.

Learning outcomes

As the result of this part of study the student receives initial skills in carrying out research and design of scientific reports and works.

Method of assessment

The defending the term paper is made by analogy with the same preliminary procedure for master dissertation. The defense is held on the Scientific seminar "Probability Theory and Mathematical Statistics". The grade depends on the quality of student's report and on results of subsequent discussion on this report. The supervisor's review of the term work also plays the big role in evaluation of the paper.

Title of the discipline:

M.3-4 Research work in a laboratory of IM SB RAS

Description

The research work in the laboratory of Probability and Mathematical Statistics of IM SB RAS aimed to consolidate and deepen the student's theoretical knowledge, to give him general culture competences and practical skills in the professional field. This skill allow graduate to work successfully in the chosen field of science. Moreover, the purpose of scientific practice is to develop the students' personal qualities that contribute to their creative activity, general cultural growth and social mobility, commitment, organization, hard work, responsibility, independence, commitment to ethical values, tolerance, perseverance in achieving the goal.

The discipline includes ongoing consultations with student's supervisor and other experts of laboratory, participation in scientific seminars and conferences, visits to scientific libraries and specialized classes of IM SB RAS.

During the scientific practice students need to:

- learn the classification systems used in science and technology (UDC, Mathematical Subject

Classification, etc.);
- train their facilities in search in bibliographic information systems;
- explore the scientific journals and monographs and the other sources of scientific and technical information, to study of the achievements of science and technology in the field;
- learn the skills of collection, processing, analysis and systematization of scientific and technical information on the subject;
- study of the safety and the rational organization of work;
- participate in research and / or implementation of technological developments;
- participate in scientific seminars.

Learning outcomes

As the result of the discipline “Research work in a laboratory of IM SB RAS” the student receives initial skills in carrying out actual theoretical research.

Method of assessment

The gradation for the discipline “Research work in a laboratory of IM SB RAS” is defined by the student's scientific supervisor. He must declare the corresponding requirements at the beginning of every training semester.

Title of the discipline:

M.3-5 Reports at scientific conferences

Description

Every year the International Student Scientific Conference “Students and the Progress in Science and Technologies” takes place in NSU (http://issc.nsu.ru/index.php?lang=1). Here students and young scientists give talks on their researches and get acquainted with the results of their colleagues. The Chair of Probability Theory and Mathematical Ststistics DMM NRU NSU organizes the sections on this conference. The students can also participate the other conferences for young scientists and make scientific reports.

Learning outcomes

As the result of the discipline “Reports at scientific conferences” the student receives initial skills in preparation and carrying out scientific reports, conducting scientific debates, asking questions, etc.

Method of assessment

The gradation for this part of training is defined by the student's scientific supervisor. The opinions of chairmen of corresponding scientific meetings are also taken into account.

M.4 FINAL STATE CERTIFICATION

Title of the discipline:
M.4-1 Passing the state exam

Description

The discipline M.4-1 "Passing the state exam" satisfy to common rules of the master state exam on DMM NRU NSU (see http://mmf.nsu.ru/education/state-exams [in Russian]). Questions and tasks for the state exam are prepared by the lecturers of core courses of the first training year of the MEP "Probability and Statistics". These courses are:

M.1-B-3 Stochastic Processes
M.1-B-6 Operations Research
M.2-C-1 Advanced Probability
M.2-C-3 Random Walks

The preliminary list of exam committee is: Prof. V.I.Lotov, Prof. I.S.Borisov, Prof. A.A.Mogulskii, Prof. D.A.Korshunov, Dr. E.A.Baklanov, Dr. A.A.Bystrov, Dr. P.S.Ruzankin.

The master state exam is carried out at the beginning of the fourth semester of the MEP "Probability and Statistics". Before the Exam, the short intensive training course on preparation to the state exam is carried out.

Learning outcomes of the discipline

As the result of the discipline "Passing the state exam", the students demonstrate knowledge and skills on the main courses of the first training year of the MEP "Probability and Statistics".

Method of assessment

The gradation for the discipline "Passing the state exam" is defined by exam committee and announced at the end of preparation course at the beginning of the fourth semester of the MEP "Probability and Statistics".

Title of the discipline:
M.4-2 Preparation and defending the master dissertation

Description

As a rule, dissertation work is the continuation of research work related with the term paper. In development of the master dissertation, the student's achievements in research are used. The master dissertation represents scientific work which includes detailed review on student's research work. The performance of the master dissertation is defined and is constantly inspected by the student's scientific supervisor.

Master's graduate work is a self-completed study which is devoted to the decision of scientific or practical problem. When working on the this problem student must demonstrate the ability and skills to solve the research problem at the high level of professional skills, the knowledge of various specific information related to the subject, the ability to report and to defend the results to the audience, to demonstrate the possession of professional competencies PC-1, PC-3, PC -6, PC-8, PC-11, PC-16.

The theme and content of the master dissertation should match the level of competency that a graduate gained in the study of core and elective disciplines of the professional cycle of MEP "Probability and Statistics".

Master dissertation is performed under the guidance of an experienced professional - researcher or Sobolev Institute of Mathematics SB RAS or Novosibirsk State University. If the supervisor is not a Chair member, curator of the Chair of Probability and Statistics is appointed. The themes of master dissertation are submitted by Chair members, by scientific supervisors of or by the student itself and are approved at the meeting of the Chair upon submission of abstracts, revealing the goals and objectives of the work. They may be based on materials of research of faculty, scientific or industrial organizations.

Master dissertation should contain abstracts part reflecting a shared professional erudition of the author, as well as independent research part configured individually or as part of the creative team on the materials collected or obtained by the student in their own period of practical training. Independent part of the master dissertation should be a completed studies that indicate a level of professional and specialized competencies of author.

It should be submitted in hard copy using scientific text editors (TeX, etc.), with the appropriate structure of the text, visual material and a bibliography, drawn up in accordance with the requirements of standards and regulations of NRU NSU.

Learning outcomes of the discipline

As the result of the discipline “Preparation and defending of the master dissertation” the student receives skills in carrying out research and design of sections of scientific reports and scientific papers, in development of final reviews on volume scientific projects and defending this review in the face of expert committees.

Method of assessment

The procedure for defending the master dissertation is the following. The “pre-defending” is realized on the Scientific Seminar “Probability Theory and Mathematical Statistics”. The recommended grade and the name of the reviewer are defined.

The final defending the Master Dissertation is carried out on the meeting State Validation Committee of NRU NSU.

The grade depends on corresponding marks given by the “pre-defending” committee and the reviewer, and also by the quality of student’s report and subsequent discussion on this report. The supervisor’s review of the term work also plays the specific role in evaluation of the dissertation.

The purpose of the defending of final qualifying work is to establish the level of professional experience of graduates to perform professional tasks in accordance with the requirements of the State Master Standard for training direction 010100 – “Mathematics”.

APPENDIX 1. Examples of admission tests and interview control questions for the MEP “Probability and Statistics”

(see http://mmf.nsu.ru/applicants/master-entexams [in Russian])

Admission test of 2012

TASK 1. Find the limit: $\lim_{x \to 0} (1+5\sin x)^{ctgx}$.

TASK 2. Find the normal Jordan form and the transition matrix of it for the matrix

$$A = \begin{pmatrix} 4 & -6 & 9 \\ 3 & -5 & 9 \\ 1 & -2 & 4 \end{pmatrix}$$

TASK 3. Find the semiaxis lengths for the intersection curve of the surface $x^2 + y^2 = 2z$ and the plane $z = 2x + 2y + 4$.

TASK 4. Let $f(x,y) = \frac{x^2 + 3y^2}{|x| + |y|}$ for $(x,y) \neq (0,0)$ and $f(x,y) = 0$ for $(x,y) = (0,0)$. Whether the function $f(x,y)$ is continuous in the point $(0,0)$? Whether it is differentiable in this point? Answers must be explained.

TASK 5. Calculate the integral: $\int \frac{dz}{(z-2)(z^{2012}+1)}$, where $\Gamma = \left\{ z = x + iy : \frac{x^2}{4} + \frac{y^2}{2} = 1 \right\}$.

TASK 6. Find the general solution of the equation: $t^2 yy'' - (2y - ty')^2 = 0$.

Admission test of 2011

TASK 1. Proof the inequality: $|\sin(\sin x) - \sin(\sin y)| \leq |x - y|$ for all $x,y \in R$.

TASK 2. For what values of the parameter p the real matrix

$$A = \begin{pmatrix} 2 & p-1 \\ 1 & p \end{pmatrix}$$

is similar to a diagonal matrix?

TASK 3. Find the centre, focuses and directrixes for the intersection curve of the cylinder $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and the plane $4x - 3z = 0$.

TASK 4. Find the limit: $\lim_{n \to \infty} \int e^{-x^n} dx$.

TASK 5. Find the residues of the function $f(z)$ in all finite singular points, which are not equal to the branch points: $f(z) = \frac{1}{2\sqrt{z} + \sqrt{3}\sqrt{z+1}}$.

TASK 6. Solve the Cauchy problem: $6tyy' - 5y^2 = -2t^2, y(1) = 3$. How many solutions there?

Examples of the Interview Control Questions

1. What is the limit of a sequence?
2. What is the limit of function in a point?
3. Define the continuous function.
4. Give the definition of the function's derivative.
5. Give the definition of the Riemann integral.
6. Give the definition of the Lebesgue integral.
7. What is the Lebesgue measure?
8. What is the Lebesgue dominated convergence theorem stated?
9. What is the sigma-algebra?
10. What is the probability?
11. What is a random variable?
11. What types of distributions of random variables do you know?
12. Define the probability density function.
13. What means the independence of random variables?
14. What is the expectation of a random variable?
15. What is the variance of a random variable?
16. What are the basic properties of variances?
17. What types of convergence(s) of random sequences do you know?
18. What is the Chebyshev inequality?
19. What is a characteristic function?
20. How to find whether the given function is a characteristic function of a distribution?
21. How one can use characteristic functions to prove limit theorems?
22. What is the weak convergence of sequences of distributions?
23. What forms of the Law of Large Numbers do you know?
24. Formulate the CLT.
25. Formulate the Poisson Theorem.
26. What methods of parameter estimation do you know?
27. What properties of samples from Gaussian distribution do you know?
28. What is the Cramér–Rao bound?
29. Formulate Rao–Blackwell theorem.
30. Formulate Neyman–Pearson lemma.